A Type-I Hybrid ARQ Protocol Over Optimal-Sequence CDMA Link

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Abstract—Recently it has been shown that the capacity of a CDMA link can be significantly enhanced by optimally selecting or adapting the set of spreading sequences employed under the constraint that a prescribed level of SNR performance is achieved. In this paper, we propose a type-I hybrid ARQ protocol which makes use of the optimally selected spreading sequences in the physical layer to allow simultaneous transmission of multiple packets over a CDMA link. Packets are divided into two classes, namely retransmission packets and newly arrived packets. Packets in each class are assigned optimally selected spreading sequences and a target SNR. With proper choices of the number of packets to be transmitted and the SNR targets in each time slot, the proposed type-I hybrid ARQ protocol provides significant improvement on the link throughput, particularly when the transmission power is abundant.

I. INTRODUCTION

Communication engineers have been striving to increase the capacity of wireless communication systems. In order to achieve this, advanced processing techniques need to be developed in both the physical and upper layers of future wireless communication systems. A promising new physical layer technique is sequence adaptation [1], [2] in code division multiple access (CDMA) systems. The main idea is to select an optimal set of spreading sequences to use in the system so that the conditions of the communication channel are matched and multiple access interference (MAI) is mitigated. Mathematically, the optimization process can be cast as the problem of choosing a set of signature sequences with minimum total power so that some pre-determined SNR targets are met [1], [2]. It is shown that the user capacity of a CDMA link can be significantly enhanced by employing the optimally selected set of spreading sequences.

In order to fully utilize the advantage of this physical layer technique, new data-link layer protocols that make good use of the optimal sequences obtained in the physical layer are needed. In particular, an efficient data-link protocol should take advantage of the fact that the target SNR of each individual packet can be adjusted (by choosing the spreading sequences optimally as described before) to increase the system throughput and to minimize the delay experienced by the packet. In this paper, we propose a type-I hybrid ARQ protocol that can integrate the usage of optimal sequences in a CDMA-based wireless network. The goal is to improve the link throughput under the constraint that a pre-determined delay performance is satisfied.

Packets are divided into two classes, namely retransmission packets and newly arrived packets. Packets in each class are assigned optimally selected spreading sequences and a target SNR. The data-link protocol is responsible to determine the target SNR's and the numbers of packets to be transmitted for the two classes in each time slot. In order to minimize delay, retransmission packets are given a higher priority in the sense that they are considered first in the resource allocation process. Any "left-over" capacity after the retransmission packets are taken care of will be used to transmit the newly arrived packets. The capacity of the link is determined by the maximum transmission power, the spreading gain, and the noise power of the system. The resource allocation function of the type-I hybrid ARQ protocol chooses the target SNR's and the numbers of packets to be transmitted for the two classes in each time slot so that the throughput of the link is maximized while the average retransmission delay experienced by a packet is guaranteed to be smaller than a pre-determined value.

The rest of the paper is organized as follows. In Sections II and III, we describe the type-I hybrid ARQ protocol at the data-link layer and the optimal-sequence CDMA link at the physical layer in detail. In Section IV, we construct the resource allocation algorithm employed by the type-I hybrid ARQ protocol. In Section V, we provide numerical results of the link throughput obtained from computer simulation to show that the proposed type-I hybrid ARQ protocol together with the resource allocation algorithm outperform an equivalent FDMA system having the same bandwidth, the same transmission power, and a similar ARQ protocol, particularly when the transmission power is abundant.

II. DATA-LINK LAYER: TYPE-I HYBRID ARQ PROTOCOL

We assume that a centralized transmitter sends packets of size $L$ bits to a number of receivers\(^1\). Synchronous direct-sequence CDMA (DS-CDMA) is employed as the underlying physical layer signaling technique. We assume that transmission time is slotted and multiple packets can be transmitted in one time slot because of the multiaccess capability provided by the CDMA technique at the physical layer.

At the data-link layer, the packet arrival process is modeled as a Poisson process, i.e., the number of packets that arrive in a time slot is Poisson distributed and the number of arrivals is independent from time slot to time slot. We assume that the trans-

\(^1\)This model describes the forward link of a wireless communication system.
mitter employs a type-I hybrid ARQ scheme [3] with a buffer that can hold a maximum of \( B \) packets. A packet arriving at the buffer is admitted to the system (stored in the buffer) if the buffer is not full at the time of arrival. Otherwise, the packet is discarded. Each packet is equipped with an error correcting code which is capable of correcting \( e \) errors in the received packet. For simplicity, we assume that the error-detecting capability required in the ARQ scheme is provided by the error correcting code and the receivers can always detect any error in a received packet. If there are more than \( e \) errors in the received packet, the receiver will send a negative acknowledgment back to the transmitter requesting for retransmission of the packet in error. Otherwise, the receiver will send a positive acknowledgment indicating that the packet is successfully received. Again for simplicity, we assume that the feedback channel is error free, the propagation delay is small, and the acknowledgments are very short. Hence the transmitter knows the status of the transmitted packets almost immediately after the transmission is completed.

Packets in the transmitter buffer are transmitted in a first-come-first-serve (FCFS) basis. If a packet is successfully transmitted in the current time slot, it will be removed from the buffer and the packets behind will shift one position up toward the front of the buffer. If a packet fails the transmission in the current time slot, it will remain in the buffer in the same position relative to the other packets before the transmission\(^2\). In this way, we can divide the packets in the buffer into two classes, namely the newly arrived packets and the retransmission packets. Because of the FCFS criterion described above, it is easy to see that the retransmission packets are always in front of the newly arrived packets in the buffer. Based on the optimal-sequence CDMA signaling technique and the resource allocation algorithm to be described in Sections III and IV, the transmitter transmits \( K_r \) retransmission packets and \( K_n \) newly arrived packets in a time slot. Moreover, the transmitter assigns different power levels to the transmitted packet so that the received SNR’s of the \( K_r \) retransmission packets and the \( K_n \) newly arrived packets are \( \gamma_r \) and \( \gamma_n \), respectively. We note that because of the FCFS criterion, all the retransmission packets in the buffer are transmitted if \( K_n \) is larger than zero in a time slot.

### III. Physical Layer: Optimal-Sequence CDMA

At the physical layer, each packet to be transmitted is assigned a spreading sequence of period \( N_r \) where \( N \) is the spreading gain. The bits in a packet are BPSK-modulated and then spread using the spreading sequence assigned. The sum of the spread signals carrying information of the packets is transmitted by the transmitter. At the receiving end, a linear MMSE receiver [4], [5] is employed to demodulate the CDMA signal.

The set of sequences (and implicitly the transmission powers of the packets) are chosen (or adapted) according to the optimal scheme described in [1] and [2]. In essence, a set of sequences that require a minimum amount of transmission power is chosen such that the target SNR’s, \( \gamma_r \) and \( \gamma_n \), of the retransmission and the newly arrived packets are achieved. Suppose that the maximum total transmission power is fixed to \( P \) and that the underlying wireless channel is modeled as an additive white Gaussian noise (AWGN) channel. Let the ratio of the total signal power to the noise power (including processing gain) be

\[
S = P/N\sigma^2
\]

and the effective bandwidth [1] of a signal with target SNR \( \gamma \) be

\[
e(\gamma) = \frac{\gamma}{1 + \gamma}
\]

Based on the results in [1] and [2], it can be shown that the following constraints are necessary and sufficient for the existence of the set of optimal sequences as described under different conditions:

(i) for \( K_r + K_n \leq N \),

\[
K_r\gamma_r + K_n\gamma_n \leq NS;
\]

(ii) for \( K_r < N \), \( K_r + K_n > N \), and \( (N - K_r)e(\gamma_r) > K_n e(\gamma_n) \),

\[
K_r\gamma_r + \frac{(N - K_r)K_n e(\gamma_n)}{N - K_r - K_n e(\gamma_n)} \leq NS;
\]

(iii) for \( K_n < N \), \( K_r + K_n > N \), and \( (N - K_n)e(\gamma_n) > K_re(\gamma_r) \),

\[
K_n\gamma_n + \frac{(N - K_n)K_re(\gamma_r)}{N - K_n - K_re(\gamma_r)} \leq NS;
\]

(iv) for \( K_r + K_n > N \) and none of the above,

\[
K_r e(\gamma_r) + K_n e(\gamma_n) \leq Ne(S).
\]

When the constraints are satisfied, the optimal choice of sequences is given by a set of (generalized) WBE sequences (see [1] for details).

The transmitter should determine suitable values of \( K_r, K_n, \gamma_r, \) and \( \gamma_n \) in each time slot so that the constraints (3)–(6) are satisfied. The choices of the parameters are obtained by a resource allocation algorithm that maximizes the throughput of the link while trying to keep the delay experienced by each packet to within a reasonable value.

### IV. Resource Allocation Algorithm

As mentioned in the previous sections, the resource allocation algorithm is responsible for selecting \( K_r, K_n, \gamma_r, \) and \( \gamma_n \) in each time slot. It selects the parameters so that the throughput of the link is maximized while the average retransmission delay\(^3\) is guaranteed to be smaller than a pre-determined value \( D_r \). By considering the retransmission delay instead of the total delay, the design of the resource allocation scheme is significantly simplified.

\(^2\)This packet may move forward in the buffer if some of the packets in front of it are successfully transmitted.

\(^3\)Retransmission delay is defined as the number of time slots elapsed from the moment that a newly arrived packet fails its first transmission to the moment that it is successfully transmitted.
A. Selection of $K_r$ and $\gamma_r$

To minimize the retransmission delay, the resource allocation algorithm assigns a higher priority to the retransmission packets in the sense that it allocates system resource to them first before considering the newly arrived packets. We recall that the constraints (3)–(6) have to be satisfied when the total transmission power is limited to $P$. Hence in some cases, no newly arrived packets can be transmitted ($K_n = 0$) and only retransmission packets are transmitted. Given $S$, it is easy to see that the maximum number of retransmission packets, $K_r$, that can be transmitted in a time slot is a function of the choice of $\gamma_r$:

$$K_r(\gamma_r) = \begin{cases} \left\lfloor \frac{N \epsilon(S)/\epsilon(\gamma_r)}{N/S/\gamma_r} \right\rfloor & \text{if } 0 < \gamma_r < S \\ \left\lfloor \frac{N S/\gamma_r} {NS/\gamma_r} \right\rfloor & \text{if } \gamma_r \geq S. \end{cases} \quad (7)$$

We can obtain an upper bound on the average retransmission delay experienced by a packet by considering the following worst case scenario:

(i) the transmitter buffer is filled completely with retransmission packets, and
(ii) the packet considered is at the end of the buffer.

Since at most $K_r(\gamma_r)$ retransmission packets can be transmitted in a time slot, this packet will not be retransmitted until the first $B - K_r(\gamma_r)$ packets in the buffer are successfully transmitted. Once this packet is transmitted, it will be retransmitted in every one of the following time slots until the transmission succeeds. Hence the retransmission delay is the sum of the waiting time and the time needed to successfully transmit the packet. It can be shown that the average retransmission delay of this worst case packet (hence an upper bound on the average retransmission delay) is given, as a function of $\gamma_r$, by

$$D_r(\gamma_r) = \max \left\{ \frac{B}{K_r(\gamma_r) P_s(\gamma_r)}, \frac{1}{P_s(\gamma_r)} \right\}, \quad (8)$$

where $P_s(\gamma_r)$ is the probability of successfully transmitting a packet given that the SNR is set to $\gamma_r$. Assuming that the MAI component at the output of the MMSE receiver is Gaussian [6] and the bit errors are independent within a packet, the packet success probability with target SNR $\gamma$ is given by

$$P_s(\gamma) = \sum_{i=0}^{L} \binom{L}{i} |P_e(\gamma)|^i (1 - P_e(\gamma))^{L-i}, \quad (9)$$

where $P_e(\gamma) = Q(\sqrt{\gamma})$ is the bit error probability. To obtain (8), we have also made the assumption that the error events are independent from packet to packet. To illustrate the result above, we consider a system with $L = 1023$, $e = 5$, $N = 31$, and $S = 15$dB. Plots of the upper bounds on the average retransmission delay versus the choice of $\gamma_r$ are given in Fig. 1 for $B = 62$, 124, and 248, respectively.

From the bound in (8), we see that the resource allocation algorithm can choose

$$\gamma_r = \inf \{ \gamma > 0 : D_r(\gamma) \leq \hat{D}_r \} \quad (10)$$

to satisfy the requirement that the average retransmission delay is bounded above by $\hat{D}_r$. For example, if $\hat{D}_r$ is set to 5 slots, then from Fig. 1, we should choose $\gamma_r = 7.9$dB and 8.4dB for the cases of $B = 62$ and 124 respectively. We note that the choice of $\gamma_r$ in (10) may not be always possible. This is indicated in Fig. 1 that no matter how large $\gamma_r$ is, $\hat{D}_r$ will be larger than 5 slots for $B = 248$. Therefore, (8) also implies an upper bound $\hat{B}$ on the buffer size $B$:

$$\hat{B} = \max \{B : D_r(\gamma) \leq \hat{D}_r \text{ for some finite } \gamma_r\}. \quad (11)$$

Once $\gamma_r$ is fixed, the choice of $K_r$ becomes implicit (because of the FCFS transmission criterion):

$$K_r = \min \{K_r(\gamma_r), k_r\}, \quad (12)$$

where $k_r$ is the current number of retransmission packets in the buffer, i.e., we should transmit as many retransmission packets as allowed by the power constraint.

B. Selection of $K_n$ and $\gamma_n$

After $K_r$ and $\gamma_r$ are chosen in a time slot, any “left-over” capacity, as specified by the constraints in (3)–(6), can be used to transmit the newly arrived packets. The values of $K_n$ and $\gamma_n$ are chosen to maximize the throughput\(^4\) of the link.

For a fixed $K_n$, the average number of successfully transmitted newly arrived packets in the current time slot is $K_n P_s(\gamma_n)$. Since the packet success probability $P_s$ is a monotone increasing function of $\gamma_n$, the throughput of the link will be maximized if we choose $\gamma_n$ to be the largest possible value satisfying the constraints (3)–(6). It can be shown that such a choice of $\gamma_n$ (as a function of $K_n$) is given as below:

\(^4\) Here throughput is defined as the average number of successfully transmitted packets per time slot.
(i) for $K_r < N$ and $1 \leq K_n \leq N - K_r$,
\[ \tilde{\gamma}_n(K_n) = \frac{1}{K_n} (NS - K_r \gamma_r); \]  
(13)

(ii) for $K_r < N$, $K_n \geq N - K_r + 1$, and $e(\gamma_r) < \frac{N - K_n}{K_r} e(S)$,
\[ \tilde{\gamma}_n(K_n) = \frac{1}{K_n} \left[ NS - \frac{(N - K_n)K_r e(\gamma_r)}{N - K_n - K_r e(\gamma_r)} \right]; \]  
(14)

(iii) for $K_r < N$, $K_n \geq N - K_r + 1$, and $\frac{N - K_n}{K_r} e(S) \leq e(\gamma_r) \leq e(S)$,
\[ \tilde{\gamma}_n(K_n) = \frac{N e(S) - K_r e(\gamma_r)}{K_n - N e(S) + K_r e(\gamma_r)}; \]  
(15)

(iv) for $K_r < N$, $K_n \geq N - K_r + 1$, and $e(\gamma_r) > e(S)$,
\[ \tilde{\gamma}_n(K_n) = \frac{1}{K_n \left( \frac{1}{N - K_r} + \frac{1}{NS - K_r \gamma_r} \right)} - 1; \]  
(16)

(v) for $K_r \geq N$ and $e(\gamma_r) < \frac{N - K_r}{K_r} e(S)$,
\[ \tilde{\gamma}_n(K_n) = \frac{1}{K_n} \left[ NS - \frac{(N - K_n)K_r e(\gamma_r)}{N - K_n - K_r e(\gamma_r)} \right]; \]  
(17)

(vi) for $K_r \geq N$ and $e(\gamma_r) \geq \frac{N - K_r}{K_r} e(S)$,
\[ \tilde{\gamma}_n(K_n) = \frac{N e(S) - K_r e(\gamma_r)}{K_n - N e(S) + K_r e(\gamma_r)}; \]  
(18)

Given $K_r$ and $\gamma_r$, the optimal number of newly arrived packets to be transmitted, $\tilde{K}_n$, in the sense of maximizing the throughput is given by
\[ \tilde{K}_n(\gamma_r, K_r) = \arg \max_K KP_\delta(\tilde{\gamma}_n(K)), \]  
(19)

where $\tilde{\gamma}_n(K)$ is defined in (13)–(18). As a result, the resource allocation algorithm chooses
\[ K_n = \min \{ \tilde{K}_n(\gamma_r, K_r), k_n \}, \]  
(20)

where $k_n$ is the current number of newly arrived packets in the buffer. Once $K_n$ is chosen, the target SNR of the newly transmitted packets is then set to be
\[ \gamma_n = \tilde{\gamma}_n(K_n). \]  
(21)

V. PERFORMANCE

We conduct computer simulations to evaluate the performance of the proposed type-I hybrid ARQ protocol and resource allocation algorithm over the optimal-sequence CDMA link. The sample system considered here has parameters $L = 1023$, $e = 5$, $N = 31$, and $S = 15$dB. The upper bound on the average retransmission delay is set to $D_e = 5$. We look at the two cases in which the buffer size $B = 62$ and 124. Plots of the throughput, packet rejection probability$^5$, average total delay, and average retransmission delay versus the packet arrival rate (load) are given in Figs. 2, 3, 4, and 5, respectively.

From Fig. 2, we see that the maximum throughput (link capacities) achieved are $32.9$ packets/slot for both $B = 62$ and 124. The throughput levels off at the maximum value after the input load exceeds the link capacity. The resource allocation algorithm limits the maximum number of packets transmitted in each time slot so that the increased load does not cause excessive congestion on the CDMA link and hence maximum throughput is maintained even when the load exceeds the link capacity. The difference between input load and link capacity translates to the fact that some newly arrived packets

$^5$Packet rejection probability is defined as the probability of the event that a newly arrived packet is rejected from entering the transmitter buffer.
are rejected from entering the transmitter buffer as indicated in Fig. 3. As expected, a larger buffer gives a smaller packet rejection probability. Moreover, it is also expected that total delay experienced by a packet will increase with the buffer size. This is verified by the results in Fig. 4. Moreover, the design of the proposed resource allocation algorithm is verified by Fig. 5 that the retransmission delays for $B = 62$ and 124 are both smaller than the pre-determined bound of 5 slots.

To conclude this section, we compare the maximum throughput achieved by the proposed type-I ARQ protocol over the optimal CDMA-link to that of a frequency division multiple access (FDMA) system with the same bandwidth and the same total transmission power. Since the spreading gain of the CDMA system is $N$, an "equivalent" FDMA system would have $N$ parallel narrowband channels, each supporting the same bit rate as that supported by a single code in the CDMA system. We assume that a standard type-I hybrid ARQ protocol [3] (at most one packet is transmitted per slot per channel) with the same error correcting capability and the same packet size as the one used in the CDMA system is independently applied on each of the $N$ parallel narrowband channels. Moreover, the power spectral density of the noise is assumed to be the same in the CDMA and FDMA systems. For the FDMA system, we can choose to evenly distribute the total transmission power over a subset of the $N$ channels. In this case, the maximum throughput achievable by the FDMA system is upper bounded by:

$$\tilde{T}_{\text{FDMA}} = \max_{1 \leq K \leq N} K P_r (N S / K).$$

Using the parameters of the CDMA system considered previously, the upper bound on the maximum throughput of the FDMA system described is 31 packets/slot. Comparing to the results in Fig. 2, we see that the proposed type-I hybrid ARQ algorithm over the optimal-sequence CDMA link outperforms the "equivalent" FDMA system. In order to fully understand the proposed type-I hybrid ARQ algorithm, we carry out the simulation when the power is limited. With $S = 5$dB and other parameter unchanged, the maximum throughput achieved are 11.8 packets/slot and 11.7 packets/slot for $B = 62$ and 124, respectively. The upper bound on the maximum throughput of the FDMA system is 11.8 packets/slot in this case. We see that the throughput of the proposed type-I hybrid ARQ algorithm approach the upper bound of the "equivalent" FDMA system when the power is limited.

VI. CONCLUSION

We have proposed a type-I hybrid ARQ protocol and an accompanying resource allocation algorithm to make use of the flexibility provided by the optimal-sequence CDMA link. From computer simulations, we have shown that the proposed type-I hybrid ARQ protocol outperforms an equivalent FDMA system having the same bandwidth, the same transmission power, and a similar ARQ protocol when the transmission power is abundant.

REFERENCES


