Doubly spread DS-CDMA for efficient blind interference cancellation

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Abstract: A new spreading scheme, which consists of two stages of spreading, is considered for DS-CDMA systems. The data of a user is spread first with a long signature sequence and then with a short signature sequence. The scheme combines the advantages of the two types of spreading techniques and allows the suppression of MAI by performing both beamforming and multiuser detection efficiently. A joint receiver with blind multiuser detection and blind beamforming is proposed for the new spreading scheme. Simulations show that the joint receiver performs better than either beamforming or multiuser detection alone. To speed up convergence, an iterative version of the joint receiver is considered. A simple adaptive sequence design method is also included.

1 Introduction

The capacity of a practical direct-sequence code-division multiple-access (DS-CDMA) system is limited by multi-access interference (MAI). To increase the capacity, two main approaches have been developed to remove or reduce MAI. The first approach is by the virtue of multiuser detection [1-4], in which the differences of the spreading sequences of the users are employed to remove MAI. Another approach to suppress MAI is to explore the spatial relationship between a user and other interferers by utilising the steering capability of an antenna array [5, 6]. These two approaches are based, respectively, on two different philosophies in spreading sequence design [7]. Short sequences of period equal to the symbol duration are assumed in most multiuser detection schemes whereas long sequences of period much larger than the symbol duration are employed in the spatial MAI rejection techniques in [5, 6]. The general shortcoming most multiuser detectors suffer from is their complexity. Although simple linear multiuser detectors have been proposed [2-4], their practicality is still uncertain. For example, the linear multiuser detector proposed in [4] is based on adaptive filtering of the received signal to reject MAI. When the order of the filter is large so that a large number of interfering signals can be suppressed, the adaptation process can be quite slow. On the other hand, the use of long sequences allows simple blind adaptive beamforming algorithms with fast adaptation to be derived [5] to reject MAI spatially. However, these spatial techniques have an intrinsic limitation. When a strong interfering signal comes from a direction close to that of the desired user signal, these methods necessarily fail. Therefore, it is desirable to incorporate some form of multiuser detection into these spatial techniques. Many attempts have been made to combine spatial processing and multiuser detection [8-10] using short sequences. Due to the difficulty involved in estimating channel parameters, such as the directions of arrival (DOAs) of the signals, these combined methods necessarily require an additional level of complexity on top of the already complex multiuser detectors.

Since the spatial MAI rejection technique considered in [5] is simple and efficient, it is another natural starting point for combining multiuser detection and spatial processing. However, the blind adaptive beamforming algorithm in [5] relies on the use of long signature sequences, which do not fit well into the realm of multiuser detection. To solve the dilemma, we consider a spectral spreading scheme that combines spreading with long sequences and spreading with short sequences. Long sequences are modelled as aperiodic random sequences while short sequences are modelled as periodic deterministic sequences. For each user, the data sequence is first spread with an aperiodic random sequence. The resultant sequence is then spread with a short periodic deterministic sequence. (In practical implementations, the two stages of spreading can, of course, be done in a single step.) The scheme combines the advantages of both spreading techniques and allows us to combine multiuser detection with spatial processing efficiently. Signal processing algorithms developed with short sequences, as well as those developed with long sequences, can be readily applied. The reason behind this convenience is that the final signal can be interpreted in two different ways: a signal spread with a short sequence or a signal spread with a long sequence. With these two simple interpretations, it can be readily seen that many algorithms for short sequences or long sequences can be applied separately. In particular, blind linear multiuser detection [4] or blind beamforming [5] can be performed. However, careful examination shows that suitable joint applications can yield superior performance. In this paper, we consider such a joint MAI cancellation algorithm based on an eigen-analysis of the received signal. To further reduce complexity and speed up convergence, we decompose the joint algorithm into iterative spatial and multiuser detection algorithms to
efficiently remove MAI. Moreover, we also present a simple sequence design method that allows efficient adaptation of the MAI cancellation algorithm to changing system load conditions.

2 System model

We describe the model of the DS-CDMA system. We assume that there are \( K \) simultaneous users in the system.

The \( k \)-th user, for \( 1 \leq k \leq K \), generates a stream of data symbols \( b^{(k)} \), given by

\[
b^{(k)} = (b_0^{(k)}, b_1^{(k)}, b_2^{(k)}, \ldots)
\]  

(1)

The data symbols \( b^{(k)} \) are random variables with \( \mathbb{E}[b^{(k)}] = 1 \). The \( k \)-th user, for \( 1 \leq k \leq K \) is provided an aperiodic random signature sequence (long sequence) \( a^{(k)} \) given by

\[
a^{(k)} = (a_0^{(k)}, a_1^{(k)}, a_2^{(k)}, \ldots, a_{N_k-1}^{(k)}, \ldots)
\]  

(2)

where the elements \( a^{(k)} \) are modelled as independent and identically distributed (i.i.d.) random variables such that \( \text{Pr}(a^{(k)} = 1) = \text{Pr}(a^{(k)} = -1) = 1/2 \). The \( k \)-th user is also provided a periodic deterministic signature sequence (short sequence) \( c^{(k)} \) of period \( N_2 \) given by

\[
c^{(k)} = (c_0^{(k)}, c_1^{(k)}, c_2^{(k)}, \ldots, c_{N_2-1}^{(k)}, \ldots)
\]  

(3)

The data sequence is spread with the aperiodic sequence to give the sequence

\[
\ldots b_1^{(k)} a_1^{(k)} b_2^{(k)} a_2^{(k)} \ldots b_0^{(k)} a_0^{(k)}
\ldots
\]

\[
b_1^{(k)} a_1^{(k)} b_2^{(k)} a_2^{(k)} \ldots b_0^{(k)} a_0^{(k)}
\ldots
\]

(4)

The resultant sequence is then spread with the periodic sequence and appropriately modulated to give the following transmitted signal:

\[
s_k(t) = \sqrt{2P_k} \sum_{i=-\infty}^{\infty} b_i^{(k)} a_i^{(k)} c_{i-N_2}^{(k)} \psi(t-iT_c) \cos(\omega t)
\]  

(5)

where the overall spreading factor \( N = N_1N_2 \), \( T_c \) is the delay between consecutive chips, \( \omega \) is the carrier frequency, \( P_k \) is the power for the \( k \)-th user signal, and \( \psi(t) \) is the chip waveform. Notice that eqn. 5 can be rewritten as

\[
s_k(t) = \sqrt{2P_k} \sum_{i=-\infty}^{\infty} \hat{b}_i^{(k)} \hat{a}_i^{(k)} c_{i-N_2}^{(k)} \psi(t-iT_c) \cos(\omega t)
\]  

(6)

where \( \hat{b}_i^{(k)} = b_i^{(k)} a_i^{(k)} \). Eqn. 6 gives the first interpretation: a signal spread with a short signature sequence at a spreading factor of \( N_2 \). Eqn. 5 can also be rewritten as

\[
s_k(t) = \sqrt{2P_k} \sum_{i=-\infty}^{\infty} \hat{b}_i^{(k)} \hat{a}_i^{(k)} \Psi_k(t-iT_c) \cos(\omega t)
\]  

(7)

where

\[
\Psi_k(t) = \sum_{i=0}^{N_2-1} c_i^{(k)} \psi(t-iT_c)
\]  

(8)

Eqn. 7 gives the second interpretation: a signal spread with a long signature sequence at a spreading factor of \( N_1 \) with a generalised chip waveform \( \Psi_k(t) \).

Without loss of generality, we consider the signal from the first user as the desired signal and the signals from all other users as interfering signals throughout the paper.

We now describe the channel model. For simplicity, we consider a multiple access channel with additive white Gaussian noise (AWGN) only. Extensions with multipath fading can be handled similarly as in [5]. An antenna array of \( D \) elements is used for signal reception. The received signal vector in complex baseband representation is given by

\[
r(t) = \sum_{k=1}^{K} \sqrt{2P_k} \left\{ \sum_{i=-\infty}^{\infty} b_i^{(k)} a_i^{(k)} c_{i-N_2}^{(k)} \psi(t-iT_c) \right\}
\times d_k + n(t)
\]  

(9)

where \( T_i \) represents the delay, \( d_k \) accounts for the overall effects of the phase shift and the DOA of the \( k \)-th user signal, and \( n(t) \) represents additive white Gaussian noise (AWGN). (The noise is also assumed to be spatially white.) We assume that synchronisation has been achieved with the first user signal. Therefore, the delay of the first user signal \( T_1 \) can be taken to be zero.

3 Spatial multiuser receivers

With the two interpretations in eqns. 6 and 7, we can construct a receiver which performs separate blind multiuser detection and blind beamforming as in [11]. A better approach is to design a receiver which can jointly perform blind multiuser detection utilising the short sequence and blind beamforming utilising the long sequence.

3.1 Joint receiver

The purpose of the long sequence is to facilitate the application of blind beamforming. It is shown in [5] that blind beamforming can be performed based on two statistics obtained at the outputs of two filters which are, respectively, matched to and orthogonal to (a segment of) the long sequence. The same idea can be applied here for the joint beamforming and multiuser detection.

\[
\text{Fig. 1 Long sequence despreading filter}
\]

First, the received signal at each antenna element is passed through the filtering structure shown in Fig. 1. In vector notation, the (vector-valued) received signal is chip-matched filtered and sampled at \( t = jT_c \) to obtain the sample:

\[
r(j) = \sum_{k=1}^{K} \sqrt{2P_k} \left\{ \sum_{i=-\infty}^{\infty} b_i^{(k)} a_i^{(k)} c_{i-N_2}^{(k)} \psi((j-i)T_c - T_k) \right\}
\times d_k + n(j)
\]  

(10)

where \( \psi(t) \) is the chip-matched filter output when the input is the chip waveform \( \psi(t) \), and \( n(j) \) is the sampled noise.
vector at the output of the chip-matched filter. The sequence \( \{ \tilde{r}_j \} \) of D-dimensional vectors is reordered to form the sequence \( \{ \hat{r}_j \} \) of \( N \)D-dimensional vectors such that
\[
\hat{r}_j = [\tilde{r}_{jN_2}, \tilde{r}_{jN_2+1}, \ldots, \tilde{r}_{j(N_1-1)+1}]^T
\]
(11)
Without loss of generality, we consider the detection of the 0th symbol. The corresponding segment of the long sequence \( \{ a_j \}_{j=0}^{N-1} \) and its orthogonal counterpart:
\[
a_j^{(1)} = \begin{cases} a_j^{(1)} & \text{for } 0 \leq j < \frac{N_2}{2} \\ -a_j^{(1)} & \text{for } \frac{N_2}{2} \leq j < N_1 - 1 \end{cases}
\]
(12)
are applied to despread the vector sequences \( \{ \tilde{r}_j \}_{j=0}^{N_1-1} \) to obtain the following two observation vectors:
\[
z = \sum_{j=0}^{N_1-1} a_j^{(1)} \tilde{r}_j = s + n + \sum_{k=2}^{K} i_k
\]
(13)
\[
\hat{z} = \sum_{j=0}^{N_1-1} a_j^{(1)} \tilde{r}_j = \hat{n} + \sum_{k=2}^{K} i_k
\]
(14)
The observation vector \( z \) contains the desired signal vector
\[
s = \sqrt{2} P_i h_0^{(1)} N_1 T_c c_0^{(1)} d_1^T, c_1^{(1)} d_1^T, \ldots, c_{N_2-1}^{(1)} d_1^T
\]
(15)
and the MAI-plus-noise component \( n + \sum_{k=2}^{K} i_k \). The observation vector \( \hat{z} \), on the other hand, contains only an MAI-plus-noise component.

Then, we employ a weight vector \( w = [w_1, w_2, \ldots, w_{DN_2}]^T \) to combine the contributions from the observation vector \( z \) to give the decision statistic
\[
D = w^H z
\]
(16)
We optimise the choice of the weight vector by maximising the signal-to-noise ratio defined by
\[
\text{SNR} = \frac{E[|w^H s|^2]}{E[|w^H (n + \sum_{k=2}^{K} i_k)|^2]}
\]
(17)
It is easy to see that the vectors \( s, n, \) and \( i_k \) are uncorrelated. Therefore, equivalently, we find the weight vector that maximises
\[
\text{SNR} + 1 = \frac{w^H R_{zz} w}{w^H R_{nn} w} = \frac{w^H R_{zz} w}{w^H R_{ww} w}
\]
(18)
where the MAI-plus-noise correlation matrix is
\[
R_{nn} = E \left[ n n^H + \sum_{k=2}^{K} i_k i_k^H \right]
\]
(19)
and the correlation matrix is
\[
R_{zz} = E \left[ z z^H \right] = E \left[ s s^H \right] + R_{ni}
\]
(20)
The second equality in eqn. 18 is due to the fact that the correlation matrix \( R_z \) of the observation vector \( z \) is the same as the MAI-plus-noise correlation matrix \( R_{nn} \). It can be shown [12] that the optimal weight vector that maximises the last expression in eqn. 18, and hence the SNR, is given by the generalised eigenvector associated with the largest generalised eigenvalue of the matrix pencil \( (R_z, R_{nn}) \).

We point out [5] that the SNR maximisation criterion above is actually equivalent to the mean squared error (MMSE) criterion considered in [4]. Hence, linear MMSE multiuser detection (based on the short sequence) is being performed implicitly by the optimal choice of the weight vector. Similar to [5], we can show that implicit beamforming is also being carried out. Moreover, since both the matrices \( R_z \) and \( R_{ni} \) can be easily estimated, respectively, from the observation vectors \( \hat{z} \) and \( z \), the joint beamforming and multiuser detection performed by this method is, in fact, blind.

### 3.2 Iterative receiver

The joint receiver considered in the previous Section suffers from the disadvantage of slow convergence (see Section 4 for details). In this Section, we decompose and perform the beamforming and multiuser detection steps iteratively in an attempt to speed up the convergence of the receiver.

![Fig. 2 Iterative beamforming and multiuser detection receiver](image)

The iterative receiver employs the two interpretations of the doubly spreading scheme described by eqns. 6 and 7 in Section 2. Blind beamforming and multiuser detection are performed iteratively utilising the results obtained from the previous iteration as shown in Fig. 2. Again, let us consider the detection of the 0th symbol. The filter in Fig. 1 is first employed to despread the received signal at each antenna element to obtain the \( N \)D-dimensional observation vectors \( z \) and \( \hat{z} \). We rearrange the elements in the vectors to obtain two \( D \times N_1 \) matrices \( Z \) and \( \hat{Z} \), respectively, in such a way that the \( (i, j) \)th element of the matrix is obtained from the \( (i + jD) \)th element of the vector. Then the following algorithm is employed to perform blind beamforming and multiuser detection iteratively to obtain a sequence of decision statistics \( \{D(n)\} \):

(i) Initialise
\[
w_{md}(1) = c^{(1)} = [c_0^{(1)}, c_1^{(1)}, \ldots, c_{N_2-1}^{(1)}]^T
\]
(ii) For \( n \geq 1 \), update:
\[
w_{bf}^{(n)} = \text{LGEV}(E[Zw^{(n)}_w(n)w^{(n)}_{md}(n)]Z^H),
\]
\[
E[\hat{Z}w^{(n)}_{md}(n)w^{(n)}_{md}(n)\hat{Z}^H]
\]
(iii) Obtain the decision statistic
\[
D(n) = w_{bf}^{(n)}(n)Zw_{md}(n)
\]
(iv) Update
\[
w_{md}(n + 1) = (E[Z^T w_{mf}^{(n)}(n)w_{bf}^{(n)}(n)Z^T])c^{(1)}
\]
(v) Increment \( n \) and go back to step 2.
In the above, the operator LGEV(·) in step 2 obtains the generalised eigenvector associated with the largest generalised eigenvalue of the matrix pencil in the argument. The symbol † in step 4 denotes the Moore–Penrose pseudoinverse of a matrix. Both the filters in steps 2 and 4 need normalisation to avoid numerical instability which may result after a large number of iterations.

The operation of the iterative receiver can be understood as follows. First, the short sequence matched filter is employed to provide the processing gain of the short sequence. Then blind beamforming is performed based on the outputs of the short sequence matched filter to achieve elementary spatial MAI cancellation. After this step, the beamforming result is reused to construct a new multuser detection filter to replace the short sequence matched filter. This step helps to remove the residual MAI that cannot be handled by spatial processing. The whole process is repeated until no more MAI can be further cancelled.

4 Performance

In this Section, we investigate the performance of the joint and iterative receivers proposed in Section 3 via Monte Carlo simulations. For simplicity, we assume that the transmissions of all the users are synchronous. The short sequences of the users are chosen randomly. Hence, the results obtained in this Section would be similar to those for the asynchronous case in which MAI is always present even if we employ orthogonal short sequences. The long sequences of the users are also chosen randomly. We assume that there are 16 active users in a sector of 45 degrees and neglect the effect of any other users outside the sector. A linear phased array of five elements is used and the overall spreading gain $N = N_1 N_2$ is set to 64. To model the near–far effect, we assume that the received powers of the interferers are log-normal distributed with the received power of the desired user as mean and a 20dB standard deviation. We also set the signal-to-white-noise ratio (SWNR) of the desired user signal to 10dB. The correlation matrices required in both the joint and iterative receivers are estimated by their corresponding time averages. The performance measure we adopt here is the average SNR of the decision statistic. The rate of convergence of a receiver means the number of observation vectors (symbols) required to obtain good enough estimates of the correlation matrices so that a certain average SNR performance can be achieved.

![Fig.3](image)

**Fig.3** Performance of joint receiver
- Solid line ($N_1 = 8, N_2 = 8, D = 5$)
- Dash line ($N_1 = 64, N_2 = 1, D = 5$)
- Dot line ($N_1 = 64, N_2 = 64, D = 1$)
- Joint receiver ($N_1 = 8, N_2 = 8, D = 5$)

First, let us compare the performance of the joint receiver with that obtained by doing beamforming or multuser detection alone. The result is shown in Fig. 3. The three configurations considered in Fig. 3 are as follows:

(i) the joint receiver proposed in Section 3.1 with $N_1 = 8, N_2 = 8, D = 5$;

(ii) the multuser detector in [4] with $N_1 = 1, N_2 = 64, D = 1$;

(iii) the beamformer in [5] with $N_1 = 64, N_2 = 1, D = 5$.

We observe from Fig. 3 that the joint receiver does outperform, as expected, both the beamformer and multuser detector alone in terms of attaining a higher average SNR after convergence. The rate of convergence of the joint receiver is faster than that of the multuser detection, but slower than that of the beamformer. This convergence order is closely related to the numbers of elements in the adaptive weight vectors of the three receivers. The smaller the number of adaptive elements, the faster is the convergence.

Next, we look at the SNR performance and the rate of convergence of the iterative receiver comparing to those of the joint receiver. The result for the configuration of $N_1 = 8$ and $N_2 = 8$ is shown in Fig. 4 where the average SNR curves obtained by four iterations of the iterative receiver are shown. We conclude from the result in Fig. 4 that the iterative receiver can attain the performance of the joint receiver with a much faster convergence rate. The complexity of the iterative receiver is of $O(D^3 + N_2^2)$, where $D$ is the number of iterations. When the number of iterations needed is small, such as in the example considered in Fig. 4, the iterative receiver presents a significant saving in complexity with respect to the joint receiver.

![Fig.4](image)

**Fig.4** Performance of iterative receiver
- (i) 1st iteration
- (ii) 2nd iteration
- (iii) 3rd iteration
- (iv) 4th iteration
- Joint receiver

5 Short sequence design

The purpose of spreading with the long sequence, besides spreading itself, is to allow easy estimation of some parameters, such as the DOA of the signal. Therefore, as long as
these parameters can be conveniently estimated, $N_1$ need not be large. On the other hand, with a larger period $N_2$ of the short sequences, more users can be differentiated in the multiuser detector stage. This is especially important when the spatial separation between the users is small. Fig. 5 demonstrates the effect with different choices of $N_2$. The system configuration considered in Fig. 5 is exactly the same as the one in the previous Section. The average SNR is obtained by using the joint receiver in Section 3.1 and 400 observation vectors are employed to estimate the correlation matrices. It can be seen that, with a larger value of $N_2$, a larger number of users can be supported for a given average SNR. However, with a larger $N_2$ (and a smaller $N_1$), a system may adapt more slowly to changes. It is because a multiuser detector with a higher order (a larger $N_2$) takes longer to settle. Therefore, the choice of $N_2$ should be a compromise between the rate of convergence and the capacity of the system.

Fig. 5 Comparison of different periods for short sequences
(i) $N_1 = 2, N_2 = 32$
(ii) $N_1 = 4, N_2 = 16$
(iii) $N_1 = 8, N_2 = 8$
(iv) $N_1 = 16, N_2 = 4$

Given a SNR performance target, one would choose the minimum $N_2$ to speed up the convergence. For example, if the target SNR is 10dB for the system considered in Fig. 5, we choose $N_2 = 4, 8, 16,$ and 32 if the number of active users is smaller than 15, 25, 45, and 55, respectively. Since the number of active users changes, this requires us to vary $N_2$ from time to time. We present here a simple short sequence design that can eliminate the need of changing the signature sequences of the users whenever $N_2$ is changed to adapt to the system load condition. For notational convenience, let us express a period of the short sequence for the $k$th user in vector form:

$$c^{(k)} = [c_0^{(k)}, c_1^{(k)}, \ldots, c_{N_2-1}^{(k)}] \quad (21)$$

Suppose $N_2$ is a power of 2. Let $N_2 = 2^l$, where $N_3$ is the smallest period of the short sequences in our design. Now, for each user, choose a sequence of $N_3$ elements:

$$d_0^{(k)} = [d_0^{(k)}, d_1^{(k)}, \ldots, d_{N_3-1}^{(k)}] \quad (22)$$

Then, construct

$$d_1^{(k)} = [c_{1,0}^{(k)}, c_0^{(k)}, c_1^{(k)}d_0^{(k)}]$$

$$d_2^{(k)} = [c_{2,0}^{(k)}, c_1^{(k)}, c_2^{(k)}d_1^{(k)}]$$

$$\vdots$$

$$d_{J-1}^{(k)} = [c_{J-1,0}^{(k)}, d_{J-2}^{(k)}, c_{J-1,1}^{(k)}d_{J-2}^{(k)}] \quad (23)$$

where all the $e_{j,s}$ are i.i.d. binary random variables as described in Section 2. Finally, we can set $d_L^{(k)}$ to be $f^{(k)}$. In this way, we can interpret the overall spreading sequence as different combinations of long and short sequences. The possible choices for the period of the short sequence are $N_3, 2N_3, \ldots, N_2$ while the corresponding long sequence spreading gain are $2^l N_3, 2^{(l+1)} N_3, \ldots, N_3$, respectively. When the number of active users in the system is about to change, the transmitter informs the receiver which interpretation of the doubly spread sequence should be used. The overhead for this side information is minimal and the actual spreading sequence is the same regardless of the interpretation at the receiver.

Fig. 6 Convergence performance of proposed sequence design method (25 users)
- $N_1 = 4, N_2 = 16$
- $N_1 = 2, N_2 = 32$

Fig. 7 Convergence performance of proposed sequence design method (55 users)
- $N_1 = 4, N_2 = 16$
- $N_1 = 2, N_2 = 32$

To verify the sequence design method described, we conduct Monte Carlo simulations to check the convergence performance when different interpretations of the sequence combinations are employed at the receiver. (We notice that the transmitted signal is the same regardless of the interpretation chosen at the receiver.) Fig. 6 shows the results for the case when there are 25 active users in the system. The joint receiver is employed in this case. The sequence is designed in such a way that the period of the short sequence can be interpreted as 16 or 32. For this load condition (25 users), we see from Fig. 6 that the 10dB SNR target can be achieved with both interpretations. If we
choose to interpret the short sequence period as 16, we reach the 10dB target in a shorter time. However, when the load increases to 55 active users as indicated in Fig. 7, we should change our interpretation, since the 10dB target cannot be achieved if we interpret the period of the short sequence to be 16.

6 Conclusions

We have considered a spreading scheme which allows us to efficiently suppress MAI by performing beamforming and multiuser detection in DS-CDMA systems. The scheme consists of two stages of spreading, which combines the advantages of long-sequence spreading and short-sequence spreading. Interference suppression algorithms developed for short sequences, as well as those developed for long sequences, can be readily applied. In particular, we have proposed a joint receiver with blind multiuser detection and blind beamforming for the new spreading scheme. Simulations show that the joint receiver performs better than either beamforming or multiuser detection alone. We have also developed an iterative receiver which can achieve the performance of the joint receiver with a much faster rate of convergence. By developing a simple sequence design method, we have shown that the double spread sequence can be made adaptive to suit changing system load conditions.

7 References