Doubly Spread DS-CDMA for Efficient Interference Cancellation

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Abstract—In this paper, a new spreading scheme, which consists of two stages of spreading, is considered for DS-CDMA systems. The data of a user is spread first with a long signature sequence and then with a short signature sequence. The scheme combines the advantages of the two types of spreading techniques and allows us to suppress MAI by performing both beamforming and multiuser detection efficiently. A joint receiver with blind multiuser detection and blind beamforming is proposed for the new spreading scheme. Simulations show that the joint receiver performs better than either beamforming or multiuser detection alone. To speed up convergence, an iterative version of the joint receiver is also considered.

Keywords—Spreading sequence design, CDMA, blind beamforming, blind multiuser detection, iterative reception

I. INTRODUCTION

The capacity of a practical direct-sequence code-division multiple-access (DS-CDMA) system is limited by multiple-access interference (MAI). To increase the capacity, two main approaches have been developed to remove or reduce MAI. The first approach is by the virtue of multiuser detection [1]–[4], in which the property of the spreading sequences of different users are employed to remove MAI. Another approach to suppress MAI is to explore the spatial relationships between a user and the interferers by utilizing the steering capability of an antenna array [5], [6]. These two approaches are based, respectively, on two different philosophies in spreading sequence design [7]. Short sequences of period equal to the symbol duration are assumed in most multiuser detection schemes whereas long sequences of period much larger than the symbol duration are employed in the spatial MAI rejection techniques in [5] and [6].

The general shortcoming most multiuser detectors suffer from is their complexity. Although simple linear multiuser detectors have been proposed [2]–[4], their practicality is still unclear. For example, the linear multiuser detector proposed in [4] is based on adaptive filtering of the received signal to reject MAI. When the order of the filter is large so that a large number of interfering signals can be suppressed, the adaptation process can be quite slow. On the other hand, the use of long sequences allows simple blind adaptive beamforming algorithms with fast adaptation to be derived [5] to reject MAI spatially. However, there is an intrinsic limitation of these spatial techniques. When a strong interfering signal comes from a direction close to that of the desired user signal, these methods necessarily fail. Therefore, it is desirable to incorporate some form of multiuser detection into these spatial techniques. Many attempts have been made to combine spatial processing and multiuser detection [8]–[10] using short sequences. Due to the difficulty involved in estimating channel parameters, such as the direction of arrivals (DOA) of the signals, these combined methods necessarily require an additional level of complexity on top of the already complex multiuser detectors.

Since the spatial MAI rejection technique considered in [5] is simple and efficient, it is another natural starting point for combining multiuser detection and spatial processing. However, the blind adaptive beamforming algorithms in [5] relies on the use of long signature sequences which do not fit well into the realm of multiuser detection. To solve the dilemma, we consider a spectral spreading scheme that combines spreading with long sequences and spreading with short sequences. Long sequences are modeled as aperiodic random sequences while short sequences are modeled as periodic deterministic sequences. For each user, the data sequence is first spread with an aperiodic random sequence. The resultant sequence is then spread with a short periodic deterministic sequence. The scheme combines the advantages of both spreading techniques and allows us to combine multiuser detection with spatial processing efficiently. Signal processing algorithms developed with short sequences, as well as those developed with long sequences, can be readily applied. The reason behind this convenience is that the final signal can be interpreted in two different ways — a signal spread with a short sequence or a signal spread with a long sequence. To see the first interpretation, notice that the sequence after the first stage of spreading can be viewed as a sequence of coded data symbols (with a repetition code changing every symbol interval). Therefore, the final signal can be viewed as the result of spreading a coded sequence with a short sequence. To see the second interpretation, notice that the second stage of spreading can be viewed as assigning a generalized chip waveform to the user. Of course, different users can now be using potentially different generalized chip waveforms. Therefore, the final signal can be viewed as the result of spreading with a long sequence.

With these simple interpretations, it can be readily seen that many algorithms for short sequences or long sequences can be applied separately. In particular, blind linear multiuser detection [4] or blind beamforming [5] can be performed. However,

1In practical implementations, the two stages of spreading can, of course, be done in a single step.
careful examination shows that suitable joint applications can yield superior performance. In this paper, we consider such a joint MAI cancellation algorithm based on an eigen-analysis of the received signal. To further reduce complexity and speed up convergence, we decompose the joint algorithm into iterative spatial and multiuser detection algorithms to efficiently remove MAI.

II. SYSTEM MODEL

We describe the model of the DS-CDMA system. We assume that there are \( K \) simultaneous users in the system.

The \( k \)th user, for \( 1 \leq k \leq K \), generates a stream of data symbols \( b^{(k)} \), given by

\[
b^{(k)} = (b_0^{(k)}, b_1^{(k)}, b_2^{(k)}, \ldots). \tag{1}
\]

The data symbols \( b^{(k)} \) are random variables with \( E[|b^{(k)}|^2] = 1 \). The \( k \)th user, for \( 1 \leq k \leq K \), is provided an aperiodic random signature sequence (long sequence) \( a^{(k)} \) given by

\[
a^{(k)} = (a_0^{(k)}, a_1^{(k)}, a_2^{(k)}, \ldots, a_{N_k-1}) \tag{2}
\]

where the elements \( a_i^{(k)} \) are modeled as independent and identically distributed (iid) random variables such that \( Pr(a_i^{(k)} = 1) = Pr(a_i^{(k)} = -1) = 1/2 \). The \( k \)th user is also provided a periodic deterministic signature sequence (short sequence) \( c^{(k)} \) of period \( N_k \) given by

\[
c^{(k)} = (c_0^{(k)}, c_1^{(k)}, c_2^{(k)}, \ldots, c_{N_k-1}). \tag{3}
\]

The data sequence is spread with the aperiodic sequence to give the sequence

\[
\cdots b_0^{(k)} a_0^{(k)} b_0^{(k)} a_1^{(k)} b_0^{(k)} a_2^{(k)} \cdots b_0^{(k)} a_{N_k-1}^{(k)} b_1^{(k)} a_0^{(k)} b_1^{(k)} a_1^{(k)} b_1^{(k)} a_2^{(k)} \cdots b_1^{(k)} a_{N_k+1}^{(k)} b_2^{(k)} a_0^{(k)} b_2^{(k)} a_1^{(k)} b_2^{(k)} a_2^{(k)} \cdots b_2^{(k)} a_{N_k-1}^{(k)} \cdots. \tag{4}
\]

The resultant sequence is then spread with the periodic sequence and appropriately modulated to give the following transmitted signal

\[
s_k(t) = \sqrt{2P_k} \sum_{i=-\infty}^{\infty} b_i^{(k)} a_i^{(k)} c_i^{(k)} \psi(t - iT_k) \cos(\omega t) \tag{5}
\]

where the overall spreading factor \( N = N_1N_2 \), \( T_c \) is the delay between consecutive chips, \( \omega \) is the carrier frequency, \( P_k \) is the power for the \( k \)th user signal, and \( \psi(t) \) is the chip waveform. Notice that (5) can be rewritten as

\[
s_k(t) = \sqrt{2P_k} \sum_{i=-\infty}^{\infty} b_i^{(k)} a_i^{(k)} c_i^{(k)} \psi(t - iT_c) \cos(\omega t) \tag{6}
\]

where \( \tilde{b}_i^{(k)} \) is given by \( \tilde{b}_i^{(k)} = b_i^{(k)} a_i^{(k)} \). Equation (6) gives the first interpretation — a signal spread with a short signature sequence at a spreading factor of \( N_2 \). Equation (5) can also be rewritten as

\[
s_k(t) = \sqrt{2P_k} \sum_{i=-\infty}^{\infty} b_i^{(k)} a_i^{(k)} \psi_k(t - iT_c) \cos(\omega t) \tag{7}
\]

where

\[
\psi_k(t) = \sum_{i=0}^{N_2-1} c_i^{(k)} \psi(t - iT_c). \tag{8}
\]

Equation (7) gives the second interpretation — a signal spread with a long signature sequence at a spreading factor of \( N_1 \) with a generalized chip waveform \( \psi_k(t) \).

Without loss of generality, we consider the signal from the first user as the desired signal and the signals from all other users as interfering signals throughout the paper.

We now describe the channel model. For simplicity, we consider a multiple access channel with additive white Gaussian noise (AWGN) only. Extensions with multipath fading can be handled similarly as in [5]. An antenna array of \( D \) elements is used for signal reception. The received signal vector in complex baseband representation is given by

\[
r(t) = \sum_{k=1}^{K} \sqrt{2P_k} \sum_{i=-\infty}^{\infty} b_i^{(k)} a_i^{(k)} c_i^{(k)} \psi(t - iT_c) \cos(\omega t) \tag{9}
\]

where \( T_k \) represents the delay, \( d_k \) accounts for the overall effects of the phase shift and the DOA of the \( k \)th user signal, and \( \eta(t) \) represents additive white Gaussian noise (AWGN).\(^2\) We assume that synchronization has been achieved with the first user signal. Therefore, the delay of the first user signal \( T_1 \) can be taken to be zero.

III. SPATIAL MULTIUSER RECEIVERS

With the two interpretations in (6) and (7), we can construct a receiver which performs separate blind multiuser detection and blind beamforming as in [11]. A better approach is to design a receiver which can jointly perform blind multiuser detection utilizing the short sequence and blind beamforming utilizing the long sequence.

A. Joint receiver

The purpose of the long sequence is to facilitate the application of blind beamforming. It is shown in [5] that blind beamforming can be performed based on two statistics obtained at the outputs of two filters which are, respectively, matched to and orthogonal to (a segment of) the long sequence. The same idea can be applied here for the joint beamforming and multiuser detection.

First, the received signal at each antenna element is passed through the filtering structure shown in Fig. 1. In vector no-
tation, the (vector-valued) received signal is chip-matched filtered and sampled at \( t = j T_c \) to obtain the sample

\[
r_j = \sum_{k=1}^{K} \sqrt{2P_k} \left\{ \sum_{i=-\infty}^{\infty} b_{i/N_2}^{(k)} q_{i/N_2}^{(k)} c_i \right\} d_k + n_j, \quad (10)
\]

where \( \hat{\psi}(t) \) is the chip-matched filter output when the input is the chip waveform \( \psi(t) \), and \( n_j \) is the sampled noise vector at the output of the chip-matched filter. The sequence \( \{r_j\} \) of \( D \)-dimensional vectors is reordered to form the sequence \( \{\tilde{r}_j\} \) of \( N_2 \)-\( D \)-dimensional vectors such that

\[
\tilde{r}_j = \left[ r_{jN_2}^{T}, r_{jN_2+1}^{T}, \ldots, r_{j(N_2-1)}^{T} \right]^{T}. \quad (11)
\]

Without loss of generality, we consider the detection of the 0th symbol. The corresponding segment of the long sequence \( \{a_j^{(1)}\}_{j=0}^{N_2-1} \) and its orthogonal counterpart

\[
\tilde{a}_j^{(1)} = \begin{cases} a_j^{(1)} & \text{for } 0 \leq j < \frac{N_2}{2}, \\ -a_j^{(1)} & \text{for } \frac{N_2}{2} \leq j < N_1 - 1, \end{cases} \quad (12)
\]

are applied to despread the vector sequences \( \{\tilde{r}_j\}_{j=0}^{N_2-1} \) to obtain the following two observation vectors:

\[
z = \sum_{j=0}^{N_2-1} a_j^{(1)} \tilde{r}_j = s + n + \sum_{k=2}^{K} i_k, \quad (13)
\]

\[
\tilde{z} = \sum_{j=0}^{N_2-1} \tilde{a}_j^{(1)} \tilde{r}_j = \tilde{n} + \sum_{k=2}^{K} i_k. \quad (14)
\]

The observation vector \( z \) contains the desired signal vector

\[
s = \sqrt{2P_0} \left[ \begin{array}{c} a_0^{(1)} d_0^{1T}, c_1^{(1)} d_1^{1T}, \ldots, c_{N_2-1}^{(1)} d_{N_2-1}^{1T} \end{array} \right]^{T}. \quad (15)
\]

and the MAI-plus-noise component \( n + \sum_{k=2}^{K} i_k \). The observation vector \( \tilde{z} \), on the other hand, contains only an MAI-plus-noise component.

Then, we employ a weight vector \( w = [w_1, w_2, \ldots, w_{DN_2}]^{T} \) to combine the contributions from the observation vector \( z \) to give the decision statistic

\[
D = w^{H} z. \quad (16)
\]

We optimize the choice of the weight vector by maximizing the signal-to-noise ratio defined by

\[
\text{SNR} = \frac{\mathbb{E} \left[ |w^{H} s|^{2} \right]}{\mathbb{E} \left[ w^{H} (n + \sum_{k=2}^{K} i_k)^{2} \right]}. \quad (17)
\]

It is easy to see that the vectors \( s \), \( n \), and \( i_k \) are uncorrelated. Therefore, equivalently, we find the weight vector that maximizes

\[
\text{SNR} + 1 = \frac{w^{H} R_{z} w}{w^{H} R_{n} w} = \frac{w^{H} R_{z} w}{w^{H} R_{x} w}, \quad (18)
\]

where the MAI-plus-noise correlation matrix

\[
R_{ni} = \mathbb{E} \left[ n n^{H} + \sum_{k=2}^{K} i_k i_k^{H} \right], \quad (19)
\]

and the correlation matrix

\[
R_{z} = \mathbb{E}[z z^{H}] = \mathbb{E}[s s^{H}] + R_{ni}. \quad (20)
\]

The second equality in (18) is due to the fact that the correlation matrix \( R_{z} \) of the observation vector \( \tilde{z} \) is the same as the MAI-plus-noise correlation matrix \( R_{ni} \). It can be shown [12] that the optimal weight vector that maximizes the last expression in (18), and hence the SNR, is given by the generalized eigenvector associated with the largest generalized eigenvalue of the matrix pencil \( (R_{z}, R_{x}) \).

We point out [5] that the SNR maximization criterion above is actually equivalent to the minimum mean squared error (MMSE) criterion considered in [4]. Hence, linear MMSE multiuser detection (based on the short sequence) is being performed implicitly by the optimal choice of the weight vector. Similar to [5], we can show that implicit beamforming is also being carried out. Moreover, since both the matrices \( R_{z} \) and \( R_{x} \) can be easily estimated, respectively, from the observation vectors \( z \) and \( \tilde{z} \), the joint beamforming and multiuser detection performed by this method is, in fact, blind.

**B. Iterative receiver**

The joint receiver considered in the previous section is conceptually simple and optimum in the sense of performing multiuser detection and beamforming jointly. However, it suffers from the disadvantage of slow convergence (see Section IV for details). In this section, we decompose and perform the beamforming and multiuser detection steps iteratively in an attempt to speed up the convergence of the receiver.

The iterative receiver employs the two interpretations of the doubly spreading scheme described by (6) and (7) in Section II.
Blind beamforming and multiuser detection are performed iteratively utilizing the results obtained from the previous iteration as shown in Fig. 2. Again, let us consider the detection of the 0th symbol. The filter in Fig. 1 is first employed to despread the received signal at each antenna element to obtain the \( N_2 D \)-dimensional observation vectors \( z \) and \( \tilde{z} \). We re-arrange the elements in the vectors to obtain two \( D \times N_2 \) matrices \( Z \) and \( \tilde{Z} \), respectively, in such a way that the \((i, j)\)th element of the matrix is obtained from the \((i + jD)\)th element of the vector. Then the following algorithm is employed to perform blind beamforming and multiuser detection iteratively to obtain a sequence of decision statistics \( \{D(n)\} \):

1. Initialize
   
   \[ w_{md}(1) = c^{(1)} = [c_0^{(1)}, c_1^{(1)}, \ldots, c_{N_2-1}^{(1)}]^T. \]

2. For \( n \geq 1 \), update
   
   \[ w_{bf}(n) = \text{LGEV}\left( E\left[ Z w_{md}(n) w_{md}^T(n) Z^H \right], \right. \]
   
   \[ \quad E\left[ \tilde{Z} w_{md}(n) w_{md}^T(n) \tilde{Z}^H \right] \).

3. Obtain the decision statistic
   
   \[ D(n) = w_{bf}^T(n) Z w_{md}(n). \]

4. Update
   
   \[ w_{md}(n + 1) = (E\left[ Z^T w_{bf}^T(n) w_{bf}^T(n) Z^* \right])^{-1} c^{(1)}. \]

5. Increment \( n \) and go back to Step 2.

In above, the operator \( \text{LGEV}(\cdot) \) in Step 2 obtains the generalized eigenvector associated with the largest generalized eigenvalue of the matrix pencil in the argument. This step performs blind beamforming based on the current multiuser detection filter \( w_{md}(n) \). The symbol \( \dagger \) in Step 4 denotes the Moore-Penrose pseudo-inverse of a matrix. This step obtains a new multiuser detection filter based on the current beamforming result. Both the filters in Steps 2 and 4 need normalization to avoid numerical instability which may result after a large number of iterations.

IV. PERFORMANCE

In this section, we investigate the performance of the joint and iterative receivers proposed in Section III via Monte Carlo simulations. For simplicity, we assume that the transmissions of all the users are synchronous. the short sequences of the users are chosen randomly. Hence, the results obtained in this section would be similar to those for the asynchronous case in which MAI is always present even if we employ orthogonal short sequences. The long sequences of the users are also chosen randomly. We assume that there are 16 active users in sector of 45 degrees and neglect the effect of any other users outside the sector. A linear phased array of 5 elements is used and the overall spreading gain \( N = N_1 N_2 \) is set to 64. To model the near-far effect, we assume that the received powers of the interferers are log-normal distributed with the received power of the desired user as mean and a 20dB standard deviation. We also set the signal-to-white-noise ratio (SWNR) of the desired user signal to 10dB. The correlation matrices required in both the joint and iterative receivers are estimated by their corresponding time averages. The performance measure we adopt here is the average SNR of the decision statistic. The rate of convergence of a receiver means the number of observation vectors (symbols) required to obtain good enough estimates of the correlation matrices so that a certain average SNR performance can be achieved.

First, let us compare the performance of the joint receiver with that obtained by doing beamforming or multiuser detection alone. The result is shown in Fig. 3. The 3 configurations considered in Fig. 3 are as below:

1. the joint receiver proposed in Section III with \( N_1 = 8 \), \( N_2 = 8 \), and \( D = 5 \);
2. the multiuser detector in [4] with \( N_1 = 1 \), \( N_2 = 64 \), and \( D = 1 \);
3. the beamformer in [5] with \( N_1 = 64 \), \( N_2 = 1 \), and \( D = 5 \).

We observe from Fig. 3 that the joint receiver outperforms both the beamformer and multiuser detector alone in terms of attaining a higher average SNR after convergence. This result is intuitive since the joint receiver combines the advantages of both the beamformer and the multiuser detector. The rate of convergence of the joint receiver is faster than that of the multiuser detection, but slower than that of the beamformer. This convergence order is closely related to the numbers of elements in the adaptive weight vectors of the three receiver. In ascending order, those numbers are 5, 40, and 64 for the beamformer, the joint receiver, and the multiuser detector, respectively. The smaller the number of adaptive elements, the faster is the con-
receiver is of $O(I(D^3 + N_2^2))$, where $I$ is the number of iterations. When the number of iterations needed is smaller, such as in the example considered in Fig. 4, the iterative receiver presents a significant saving in complexity with respect to the joint receiver.

V. CONCLUSIONS

We have considered a spreading scheme which allows us to efficiently suppress MAI by performing beamforming and multiuser detection in DS-CDMA systems. The scheme consists of two stages of spreading. For each user, the data sequence is first spread with a long signature sequence. The resultant sequence is then spread with a short signature sequence. The scheme combines the advantages of the two types of spreading techniques. Interference suppression algorithms developed for short sequences, as well as those developed for long sequences, can be readily applied. In particular, we have proposed a joint receiver with blind multiuser detection and blind beamforming for the new spreading scheme. Simulations show that the joint receiver performs better than either beamforming or multiuser detection alone. We have also developed an iterative receiver which can achieve the performance of the joint receiver with a much faster rate of convergence.

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