April 7' 2004

University of Florida
Wireless Information Networking Group

Surenra Boppanna

Capacity Regions for Wireless Ad Hoc Networks
Routing, spatial re-use and successive interference cancellation techniques on network performance, including power control, multihop transmission protocols, which may be sub-optimal, define capacity regions as the maximum achievable rates under specific Shannon capacity region of Ad-hoc networks remains an open problem.

Capacity of Wireless Adhoc Networks
transmitter, receiver and an infinite buffer

Ad hoc network with $n$ nodes $A_1, A_2, \ldots, A_n$ each equipped with a

System Model
The SIR at node $A_j$ is given by

$$\frac{\sum_{k \in T, k \neq j} C_k P_k}{C_j P_j} + M_j = \gamma_j$$

Let $\mathcal{A}$ be the set of transmitting nodes at a given time, each node $A_j$ transmitting with power $P_j$

\{$u_1, u_2, \ldots, u_J$\} is the AWGN vector and $H$ is the channel gain matrix

$\mathcal{C}$ is the power vector, $P$ is the power matrix

System Model (2)
Perfect knowledge of $p, H$, and the transmission schedule is assumed.

Specific modulation scheme.

The maximum data rate that satisfies a given BER requirement under a given Shannon's capacity or it might reflect.

$(\gamma, \lambda_f) (\gamma, \lambda_f) (\gamma, \lambda_f) (\gamma, \lambda_f)$

The nodes $A_i, A_j$ agree on transmission rate $R_f$.  

System Model(3)
node of the transmitted information
and for each of these pairs the transmission rate and the original source
list of all transmit-receive node pairs in operation at that time,

\[ V_{A3} \rightarrow V_{A1} \]

flow between different nodes in a network at a given instant

A transmission scheme $S$ is a complete description of the information

Transmission Schemes
We say that network is using the time division scheduling if 

\[
\text{\textit{S}} \subseteq \bigcup_{i=1}^{n} a_i \bigcup_{\gamma=1}^{\gamma} T
\]

\( S \) operating for a fraction of the frame \( a_i \) with

\( S \) within each frame the network operates using successfully \( S \) within

some fixed duration

\[ \text{of} \]

\[ \text{frames of} \]

\[ \text{Time division schedules} \]
\[ T = 0.75s^1 + 0.25s^2 \]

**Time Division Scheduling**

FIG. 2 shows two possible time division schedules with \( T = 0.75s^1 + 0.25s^2 \) and \( T = 0.75s^1 + 0.25s^2 \).
The data

Row index of a rate matrix entry corresponds to the original source of

\[
\begin{cases}
0, & \text{otherwise} \\
\mathbf{H}, & \text{transmits information with } \mathbf{H}^\top \text{ as the original information source}
\end{cases}
\]

\[
\mathbf{H}, & \text{receives information with } \mathbf{H}^\top \text{ as the source}
\]

\[
\mathbf{R}\left(\mathbf{S}\right) = \mathbf{R}
\]

Scheme:

Rate Matrices

Rate Matrices
Then its rate matrix is \( R = I_n \) if and only if \( 0 < a_i \leq 1 \) with \( a_i \) entries.

If \( a_i = 1 \) is a time division schedule, then

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
10 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 10 & 0 & 0 \\
0 & 0 & 10 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 10 & 0 \\
0 & 0 & 0 & 10 & 0
\end{bmatrix}
= R
\]

\( R \) has all non-negative entries, and every row sums to zero.

Elements in a row sum to zero.

Data received

Negative entry corresponds to data forwarded and positive entry indicates received.

Column index specifies the receiver of the transmitter of the data.
\[ u \cup \left\{ I = \arg \min_{a \in \mathcal{R}^q \cap \mathbb{R}^q} \sum_{N=1}^{l} a^T \mathbf{R}_N a : 0 \preceq a \preceq a^T \right\} \] = u \cup (\{ R^q \})^c \equiv (\{ R^q \})^c \subset C = C

Matrix of non-negative off-diagonal entries
- Capacity region is defined as the convex hull of all the basic rates
- Matrices with non-negative off-diagonal entries not allowed
- Weighted sum of all basic rates matrices of this protocol of a network under time division and a given protocol is the set
- Capacity of a network under time division and a given protocol
- Basic rate matrices

Equivalent schemes or equivalently a set of rate matrices corresponding to a given transmission protocol we have a collection of

Ad Hoc Capacity Regions
source-destination pairs.

$\text{off-diagonal elements equal to } R$ and $u(n - 1)$ is the total number of
instances equal to $R$, where $R_{\text{max}}$ is the largest $R$ for all the
rate

If all nodes wish to communicate with all other nodes, at a
common
given transmission protocol is the maximum aggregate communication
uniform capacity $C_n$ of a network under time division routing and a

The capacity region is a subset of a $u(n - 1)$ subspace

and transmission protocol

The shape of the capacity region depends on the topology, parameters

Capacity Regions
The capacity region is given by:

\[ u \cup \{v \mid y = \frac{1}{\rho} R_x v \} = C_0 \]

\[ C \]

Capacity regions for various transmission protocols
\[
\mathcal{D} \cup \{e \cdot N \cdots \cdot \{e \cdot \mathcal{H}\}^i x^{(e, \mathcal{H})} \}^i \mathcal{C} = e \mathcal{C}
\]
\[
1 + \frac{i^n}{\sum_{\frac{z=i}{z}} (I + \frac{z}{u} - u)(I - u)u} = e \mathcal{N}
\]

- Multiple Routing with Spatial Reuse
- Multiple Routing, No Spatial Reuse

Capacity Regions for Various Transmission Protocols (2)
\[ I + \left( \sum_{i=1}^{\lfloor \frac{c}{u} \rfloor} i^2 \right) \\mathcal{C}_1 d^i u \frac{i^2}{(1 + \frac{c}{u} - u) \cdot (1 - u) u} = \mathcal{N} \]

Successive Interference Cancellation

\{ i d \ldots \frac{d}{d^2} \ldots \frac{d}{d^i} \}: \text{ where node } i \text{ transmits at one of possible power levels:}

\[ u \mathcal{D} \cup \left\{ p \mathcal{N} \ldots i = \sum_{i=1}^{\lfloor \frac{c}{u} \rfloor} \frac{i^2}{(1 + \frac{c}{u} - u) \cdot (1 - u) u} \right\} = p \mathcal{C} \]

\[ I + \left( \sum_{i=1}^{\lfloor \frac{c}{u} \rfloor} i^2 \right) \\mathcal{C}_1 d^i u \frac{i^2}{(1 + \frac{c}{u} - u) \cdot (1 - u) u} = \mathcal{N} \]

Power Control

Capacity Regions for Various Transmission Protocols(3)
\[
\cup_{u \in \{N^2, R^2, \ldots \}} C = \bigcup_{u \in \{N^2, R^2, \ldots \}} C = \sigma C
\]

\[
1 + (1 - i \cdot z) \ast \ldots \ast 1 + (1 - i \cdot z) \ast 2 + (1 - i \cdot z) \ast \ldots \ast 1 = \sigma C
\]

Where
Relativistic performance of transmission protocols
\[ \sum_{i=1}^{N} x_i \geq \ln \left( \frac{1}{\text{Rate}} \right) \]

subject to:

\[ 0 \leq x_i \leq 1 \]

minimize:

\[ \sum_{i=1}^{N} (x_i) \]
Gives significant speed gains

minimal set of rate matrices that sufficiently describes the capacity region

Since not all matrices contribute to the capacity regions, choosing the

interactable for large networks

As the number of basic rate matrices increases, computations become

matrices have to be computed

In order to compute the capacity region, the set of all the basic rate

capacity regions

By iteratively solving the LP we can determine boundary points of the

capacity region

If the problem is feasible with $\bar{g}(\hat{d}, x) \leq 1$, then $R$ belongs to the

Computational Issues (2)
also reproduce most of the capacity region. a "good enough" subset of rate matrices, that will be manageable and

One possible future work could be developing methods for determining

limited or no rate adaptation is used. significant gains, but gains from power control are significant only if very

It was shown that multihop routing, spatial re-use and SIC all lead to

was developed. network under time division routing and a given transmission protocol

A mathematical framework for finding the capacity region of an ad hoc

Conclusions