LINK DISTANCE-BASED NODE ACTIVATION FOR GEOGRAPHIC TRANSMISSIONS IN FADING CHANNELS

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***Abstract*—We consider a collection of battery-operated, low-power sensors randomly deployed within a geographic region for the purpose of sensing/monitoring the environment. We assume that a transmitting sensor broadcasts messages to receiver nodes over a randomly varying(fading) wireless channel. With the intention of maximizing overall network lifetime in such a setting, we propose a protocol that allows the receivers to sleep (power off) most of the time and turn on (activate) according to a function that depends on their link distance from the transmitter. We first derive the optimum density of link distances assuming N (fixed) nodes to be awake such that the distance to the farthest successful receiver is maximized. We then extend our analysis to networks where the number of nodes are distributed according to the Poisson distribution. In particular, we utilize this optimal density function to derive a simple expression for the conditional probability that a node turns on, given its distance from the transmitter. We also derive the minimum node density and scaling constant that meets the constraint on the average number of nodes that are awake to listen to a transmission. We compare the performance of our protocol with a simpler protocol that turns on all the nodes around the transmitter upto a certain distance. For fairness in comparison, the average number of nodes that activate is kept the same in both the protocols.**

I. INTRODUCTION

Wireless sensor networks(WSNs) have made a huge impact in commercial as well as military applications. These networks combine simple, low-cost communications equipment, minimal computation requirements and some sort of sensing capability of the physical environment into a single intelligent device to form a *wireless sensor node* or simply, a *node*.

Despite the widespread deployment of sensor networks, they suffer from a serious disadvantage - *the sensors are limited in battery life*[1]. As a result, maximizing network lifetime is a key component in the design of communication protocols for sensor networks. Thus, system protocols are designed that provide energy savings by allowing nodes to be turned on (activated) in a controlled manner and remaining switched off at other times. Such protocols are called *sleeping protocols* in the literature [2], [3]. For example, sleeping protocols have been designed for which sensors located very far from a transmitter go to sleep (remain turned off) [4] while the sensors closer to the transmitter stay awake.

The energy benefit of routing over longer hop lengths was shown in [5] by comparing several routing strategies in a Rayleigh fading channel. Geographic routing in wireless networks has been reported in [6] and [7] where the authors propose Geographic Random Forwarding (GeRaF) – a multihop packet forwarding scheme that chooses the node that is closest to the destination as the next-hop neighbour. In GeRaF, the authors consider that there is a specific destination node and that the transmitter is aware of its location. In [7] the authors discuss the energy management issues of GeRaF in a fading channel. We have shown in [8] that multiuser diversity can be utilized to increase the per-hop transmission distance in a geocasting scenario. In [9], the authors use a stochastic-decision framework to determine the number of nodes to activate at any given time such that a global utility function is maximized. Most of the previous work on power savings in geographic transmission schemes for sensor networks assumed that sensor nodes are always on. We have a different consideration – we assume that the nodes themselves decide whether to turn on/off based on their distance from the transmitter. This paper is organized as follows. The system model is described in Section II, and our analysis is presented in Section III. We discuss our results in Section IV, and summarize our conclusions in Section V.

II. SYSTEM DESCRIPTION

A. Transmission Model

We consider broadcast transmissions from a source node(transmitter) to randomly located destination nodes(receivers) in a geocasting environment. In such a scenario, a prespecified destination node does not exist and the objective of the transmitter is to disperse the message as far as possible into the network. To simplify the analysis, we consider a single-hop scenario and ignore the effect of packet retransmissions in our model. In addition to propagation losses due to distance, we consider the wireless channel between the transmitter and each of the receivers to be randomly varying according to some generalized fading distribution. We assume the fading process to be flat over the frequency range of transmission and relatively slow in comparison to the data rates of transmission. We also assume that the channel fading gains do not vary during each time-slot during which a message is broadcast from the transmitter, and we assume that the fading gains are independent between nodes. We also assume the transmitter to transmit at unit power. Thus, the signal power received at an arbitrary receiver depends on the distance between the transmitter and that receiver and the fading gain at the receiver during that transmission.

The location of the nodes is represented as a homogeneous Poisson point process, $\{X_i\}$ in $\mathbb{R}^2$ with intensity λ per unit area. Here we have used X_i to denote the random link distance from the transmitter to receiver i. Let H_i denote the square of the channel gain at receiver i. Without loss of generality, we normalize all distances to the unit radius of the AWGN channel and hence the instantaneous SNR at receiver i can be modelled as

$$\gamma_i = H_i X_i^{-n}, \quad (1)$$

where n denotes the path-loss exponent. We assume a wireless network having low-traffic load, so that we can easily ignore the impact of interference on the received SNR γ_i. We assume that all the sensors are equipped with identical, omni-directional antennas. We consider transmissions within an annular ring with inner radius R_1, outer radius R_2, as shown in Fig. 1. The probability that there are l nodes inside this annulus is given by

$$P[\aleph = l] = \frac{\exp(-\lambda B)(\lambda B)^l}{l!}, \; l = 0, 1 \ldots, \infty \quad (2)$$

where, $B = \frac{\theta}{2\pi}\pi(R_2^2 - R_1^2)$ is the area within which the nodes are located. Here, we address the scenario where $\theta = 2\pi$. Although not shown in this paper, it is straightforward to extend our analysis for any general θ, $(0 < \theta \leq 2\pi)$ without significant change in the results.

Under the Poisson point process, the nodes within such a region are distributed uniformly in area, and hence the distribution function for the distance to an arbitrary receiver assuming that the source radio is at the origin is given by

$$F_{X_i}(x) = \begin{cases} 0, & x \leq R_1 \\ \frac{x^2 - R_1^2}{R_2^2 - R_1^2}, & R_1 < x \leq R_2 \\ 1, & x > R_2. \end{cases} \quad (3)$$

III. Analysis

A. Optimum Node Activation

Our goal is to design a distance-aware sleeping protocol that achieves the maximum transmission distance under the constraint on the average number of nodes that are awake to receive a transmission. For that purpose, we define node activation in the following manner.

Let ζ_i denote the event that node i located at distance X_i activates, i.e. node i is awake. Here we have used X_i to denote the random link distance from the transmitter to receiver i. We define $\psi(x)$, the node activation probability, as $\psi(x) \triangleq P(\zeta_i = 1 | X_i = x)$. Thus, conditional on X_i, the Bernoulli random variable, ζ_i can be written as,

$$\zeta_i = \begin{cases} 1, & \text{with probability } \psi(x), \\ 0, & \text{with probability } (1 - \psi(x)). \end{cases} \quad (4)$$

Note that the node activation probability $\psi(x)$ is identical for all the nodes and hence does not depend on i. We wish to obtain the optimal function $\hat{\psi}(X_i)$ that determines when node i activates in order to maximize the expected transmission distance to the farthest receiver who has successfully decoded the message, subject to a constraint on the expected number of nodes that activate. An arbitrary node is able to decode a

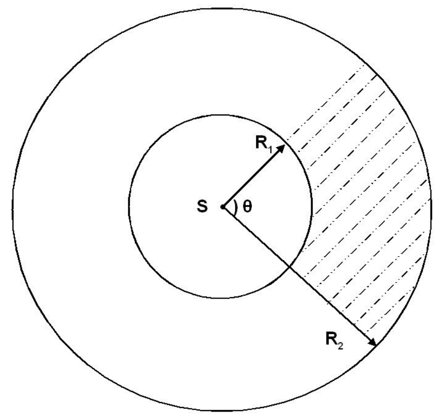

Fig. 1. Node activation region : source S at origin, nodes located inside the annular ring with inner radius R_1 and outer radius R_2

message successfully if it is awake and the received SNR is greater than some threshold κ, which is assumed to be identical for all receivers. Thus, the distance to a successful receiver V_i can be expressed as,

$$V_i = \begin{cases} X_i, & \gamma_i > \kappa \;\; \cap \;\; U_i < \psi(X_i) \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

Using (1) in (5) and simplifying, we have,

$$V_i = \begin{cases} X_i, & X_i < \sqrt[n]{H_i \kappa^{-1}} \;\; \cap \;\; U_i < \psi(X_i) \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

Here U_i is a uniformly distributed random variable on $[0, 1]$ and the channel gains H_i are distributed according to some distribution function. Note that X_i, H_i and U_i are mutually independent random variables. Then, conditioned on $\aleph$ active nodes in the network, the distance to the farthest successful receiver located at distance V_{max} from the transmitter can be expressed as

$$V_{max} = \begin{cases} \max\left\{V_1, V_2 \ldots V_\aleph\right\}, & \aleph = 1, 2 \ldots \\ 0, & \aleph = 0. \end{cases} \quad (7)$$

We wish to find the optimum $\hat{\psi}(X_i)$ such that $\mathbb{E}[V_{max}]$ is maximized, subject to a constraint on the expected value of $\aleph$. Thus, we can express $\hat{\psi}(X_i)$ as,

$$\hat{\psi}(X_i) = \arg\max_{\psi(X_i)} \mathbb{E}[V_{max}]$$

such that the expected number of nodes that are awake is a constant, i.e. $\mathbb{E}[\aleph] = \mu$, where μ is some constant. We note here that obtaining the optimal node activation function $\hat{\psi}(x)$ as described above is a difficult problem to solve because $\aleph$ is a random variable. Hence, we have simplified the above problem in the following manner.

B. Pseudo-optimum protocol

Let us deterministically fix the number of nodes inside the shaded region in Fig. 1 that are awake to listen to a transmission. We denote that number as N. We first derive the optimal density of link distances for N active nodes and then obtain the node activation probability ψ that achieves this density and for which $\mathbb{E}[\aleph] = \mu$ holds.

For the sake of clarity, we make use of the following transformation of the channel gains H_i, i.e., denote, $Y_i = \sqrt[n]{(H_i\kappa^{-1})}$. So, $Y_1, Y_2, \ldots, Y_N$ form a sequence of N positive random variables that are i.i.d. Let us denote the distribution function of Y to be $\widetilde{G}(y)$ and the density as $\tilde{g}(y)$. Now, we can redefine V_i from (6) as,

$$V_i = \begin{cases} X_i, & X_i < Y_i \\ 0, & \text{otherwise.} \end{cases}$$

Now we can express $F_V(t)$ as,

$$\begin{aligned} F_V(t) &= \mathbb{P}(V_i \le t,\ X_i < Y_i) + \mathbb{P}(V_i \le t,\ X_i \ge Y_i) \\ &= \mathbb{P}(X_i < Y_i,\ Y_i \le t) + \mathbb{P}(X_i \le t,\ Y_i > t) \\ &\quad + \mathbb{P}(X_i \ge Y_i) \end{aligned}$$

After some simplifications, $F_V(t)$ may be obtained as,

$$F_V(t) = 1 - \int_t^\infty \left[1 - \widetilde{G}(u)\right] dF(u). \tag{8}$$

We can write the expectation of V_{max} as,

$$\mathbb{E}[V_{max}] = \int_0^\infty 1 - F_{V_{max}}(t)\, dt. \tag{9}$$

Since $V_1, V_2, \ldots, V_N$ are i.i.d., the distribution of V_{max}, denoted as $F_{V_{max}}(t)$ can be expressed as

$$F_{V_{max}} = [F_V(t)]^N. \tag{10}$$

Applying (8) and (10) to (9), we have,

$$\mathbb{E}[V_{max}] = \int_0^\infty 1 - \left[1 - \int_t^\infty \left(1 - \widetilde{G}(u)\right) dF(u)\right]^N dt. \tag{11}$$

We note again that our goal is to solve the following optimization problem,

$$\hat{F} = \arg\max_F \ \mathbb{E}[V_{max}]$$

where $\mathbb{E}[V_{max}]$ is given in (11). Before going on to obtain the optimal distribution function $\hat{F}$, we first need the following result to establish the existence and uniqueness of $\hat{F}$.

Proposition. *A maximizing distribution, $\hat{F}$ exists and is unique in the set of all distribution functions F.*

Proof: The convex set of probability measures on $[0, \infty]$ is compact in the topology of weak convergence. (Note that ∞ is included). From (11), we see that the map, $dF \to \mathbb{E}[V_{max}]$ is strictly concave if $N > 1$. It is not difficult to show that it is continuous in the weak topology. Therefore, there is a unique distribution F for which $\mathbb{E}[V_{max}]$ is maximized. ■

Let us denote $g(u) = 1 - \widetilde{G}(u)$ and let F maximize

$$\int_0^\infty 1 - \left[1 - \int_t^\infty g(u)dF(u)\right]^N dt \tag{12}$$

In order to find an expression for the optimal distribution function F, we need to investigate some of its properties. We start by imposing some regularity conditions on F with the help of the following hypothesis,

Hypothesis 1.

$$\lim_{s\to\infty} g(s) = 0 \quad \text{and} \quad \limsup_{s\to 0} g(s) < \infty$$

Define the function $\Omega(s)$ as follows,

$$\Omega(s) = g(s) \int_0^s \left[1 - \int_t^\infty g(u)\, dF(u)\right]^{N-1} dt. \tag{13}$$

Applying this hypothesis, we have,

$$\lim_{s\to\infty} \Omega(s) = 0 \text{ and } \lim_{s\to 0} \Omega(s) = 0.$$

We have shown in the Appendix that dF is concentrated over some set $\mathcal{A}$. (Note that this condition does not guarantee that the optimal density exists. We formally show that the optimal density exists and equals dF with the help of Hypothesis 2.) Consequently, there is a finite interval $[\alpha, \beta]$ with $\alpha \neq 0$ such that dF is concentrated on $\mathcal{A}$. Let $C^{N-1} = \max_s \ \Omega(s)$. Then clearly, $\mathcal{A} = \left\{ s : \Omega(s) = C^{N-1} \right\}$. So for each $s \in \mathcal{A}$,

$$\int_0^s \left[1 - \int_t^\infty g(u)\, dF(u)\right]^{N-1} dt = \frac{C^{N-1}}{g(s)} \tag{14}$$

Note that in (14), both F and C are unknown. Let us assume for the moment that $\mathcal{A}$ is an interval $[\alpha, \beta]$. Differentiating (14) leads to

$$1 - \int_s^\beta g(u)dF(u) = C\left[\frac{-g'(s)}{(g(s))^2}\right]^{\frac{1}{N-1}} \tag{15}$$

for every $\alpha \le s \le \beta$. Let us now make the following substitution,

$$\rho(s) = \left[\frac{-g'(s)}{(g(s))^2}\right]^{\frac{1}{N-1}}. \tag{16}$$

Using integration by parts, we can rewrite (15) as,

$$1 + \int_s^\infty g'(u)F(u)du + F(s)g(s) = C\rho(s) \tag{17}$$

Hypothesis 2. *g is twice continually differentiable and bounded away from zero on compact sets.*

Under this hypothesis, (17) shows that F is differentiable, so that $dF(u) = f(u)du$. Using this in (15) and differentiating, we get,

$$f(s) = C\frac{\rho'(s)}{g(s)}, \quad \alpha \le s \le \beta \tag{18}$$

Note that we have still not identified C, α and β in (18). Substituting (18) into (15), and simplifying, we have,

$$C\rho(\beta) = 1. \tag{19}$$

We also have $\int_\alpha^\beta dF(u) = 1$, which implies,

$$C\int_\alpha^\beta \frac{\rho'(s)}{g(s)} ds = 1. \tag{20}$$

Recall our hypothesis that $\mathcal{A} = [\alpha, \beta]$. For $\alpha \le s \le \beta$, we can rewrite (14) as,

$$\int_0^s \left[1 - \int_{t\vee\alpha}^\beta g(u) f(u) du\right]^{N-1} dt \quad = \quad \frac{C^{N-1}}{g(s)}. \tag{21}$$

Using (16) in (21) and then applying (19), we have for $\alpha \le s \le \beta$,

$$\alpha [C\rho(\alpha)]^{N-1} + \int_\alpha^s C^{N-1}\rho(t)^{N-1} dt = \frac{C^{N-1}}{g(s)}. \tag{22}$$

Using the definition of ρ from (15), substituting in (22) and simplifying, we have for $\alpha \le s \le \beta$,

$$\alpha (-g'(\alpha)) = g(\alpha) \tag{23}$$

Under reasonable conditions on g, this determines α uniquely and from (19) and (20), C and β can be uniquely determined for any generalized distribution of the fading gains.

In order to obtain a simple expression for the activation function, $\psi(x)$ for the pseudo-optimum protocol, we revert back to our optimum node-activation problem. We assume that the conditional distribution of the link distance to receiver i, given that receiver i is active, achieves the optimum distribution function. That is, we equate, $F_{X_i}(x|\ \zeta_i = 1) = \hat{F}(x)$. Applying the Bayes rule of conditional probability, we obtain,

$$\psi(x) = \frac{\hat{f}_{X_i}(x)}{f_{X_i}(x)} \int_0^\infty \psi(u) f_{X_i}(u)\, du \quad \textbf{a.e.}$$

We note here that the above equation has more than one solution for $\psi(x)$. But since we have defined $\psi(x)$ as a probability, we obtain a particular solution for ψ as,

$$\widetilde{\psi}(x) = \frac{1}{\widetilde{M}} \frac{\hat{f}(x)}{f_{X_i}(x)}, \tag{24}$$

where $\widetilde{M} = \max_{R_1 < x \le R_2} \left\{ \frac{\hat{f}_{X_i}(x)}{f_{X_i}(x)} \right\}$

We also find the minimum node density $\widetilde{\lambda}$ that satisfies the constraint on the average number of active nodes μ, with each node having activation probability $\widetilde{\psi}(x)$. Equating the expected number of nodes that activate inside the annulus $[R_1, R_2]$ to μ, we have,

$$\widetilde{\lambda}\pi \left(R_2^2 - R_1^2\right) \int_{R_1}^{R_2} \widetilde{\psi}(x) f_X(x) dx = \mu \tag{25}$$

Using (24) in (25), we obtain

$$\widetilde{\lambda} = \frac{\mu \widetilde{M}}{\pi \left(R_2^2 - R_1^2\right)}.$$

Therefore, for an arbitrary node density $\lambda \ge \widetilde{\lambda}$, the constant scaling factor M for the function $\psi(x)$ can be expressed in terms of $\widetilde{\lambda}$ as, $M = \widetilde{M}\frac{\lambda}{\widetilde{\lambda}}$ and hence the node activation probability $\psi(x)$ would then scale as

$$\psi(x) = \frac{1}{M} \frac{\hat{f}(x)}{f_{X_i}(x)}. \tag{26}$$

Note that the activation function $\psi(x)$ found in this way is not necessarily optimal in the sense of maximizing the distance to the farthest successful receiver. Nevertheless, this approach allows us to derive a simple analytical expression for the node activation function that would help in activating nodes intelligently based on their distance from the transmitter.

C. Rayeligh channel model

We now provide an application of the results derived in the previous section to the specific case when the channel gains are distributed according to the Rayleigh distribution and the nodes are located inside a region given by Fig. 1. In other words, the square of the channel gains, i.e. $\{H_i\}$ are i.i.d. exponential random variables with normalized mean 1. So, we have the distribution of the random fading gains, $\tilde{G}$ given by, $\tilde{G}(y) = 1 - \exp(-y)\,, \quad y > 0$. Hence,

$$g(y) = 1 - \tilde{G}(y) = \exp(-y)\,,\ y > 0. \tag{27}$$

Using (27), (16), (19), (20) and (23), we finally obtain the optimum density of link distances $\hat{f}(x)$ for $R_1 < x \le R_2$ as,

$$\hat{f}(x) \quad = \quad \frac{1}{(N-1)R_2^{\frac{n-1}{N-1}} \exp\left(\kappa\frac{R_2^n}{N-1}\right)} x^{\frac{n-1}{N-1}} \cdot \exp\left(\kappa\frac{N}{N-1}x^n\right)\left[\frac{n-1}{x} + n\kappa x^{n-1}\right] \tag{28}$$

Here,

$$R_1 = \alpha = \sqrt[n]{(n\kappa)^{-1}}, \tag{29}$$

and we have obtained the value of $R_2 = \beta$ numerically by solving the integral equation,

$$\int_{R_1}^{R_2} \hat{f}(x) dx \quad = \quad 1. \tag{30}$$

For the Rayleigh channel, we obtain the node activation probability $\psi(x)$ in the following manner. We first differentiate (3) to obtain the density $f_X(x)$ and then substitute $f_X(x)$ alongwith the optimum density $\hat{f}(x)$ obtained from (28) in (26). Thereafter, for $R_1 < x \le R_2$ we obtain $\psi(x)$ as

$$\psi(x) \quad = \quad \frac{\left(R_2^2 - R_1^2\right)}{2(N-1)R_2^{\frac{n-1}{N-1}} \exp\left(\kappa\frac{R_2^n}{N-1}\right)} x^{\frac{n-N}{N-1}} \cdot \exp\left(\kappa\frac{N}{N-1}x^n\right)\left[\frac{n-1}{x} + n\kappa x^{n-1}\right] \tag{31}$$

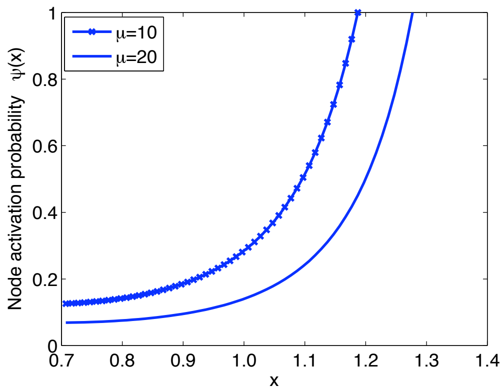

Fig. 2. Node activation probability $\psi(x)$ with nodes located within the annulus, subject to a constraint on the expected number of active nodes μ.

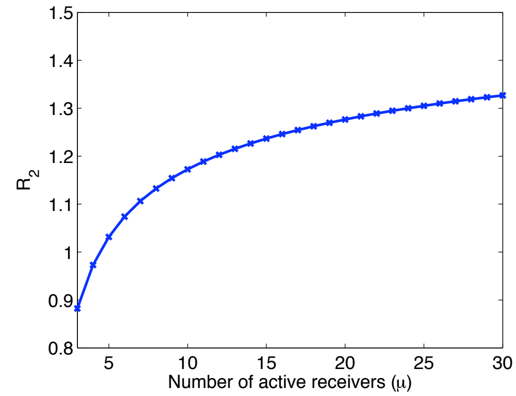

Fig. 3. Outer radius R_2 for the pseudo-optimum protocol plotted vs. the expected number of active nodes μ.

IV. RESULTS

In this section, we evaluate the performance of our pseudo-optimum protocol in a Rayleigh fading channel with path-loss exponent $n = 4$ which represents a typical urban environment with dense multipath. We assume the threshold $\kappa = 1$ at all the receivers. We consider the pseudo-optimum protocol turns on nodes geographically located inside the annulus of Fig. 1. The expected number of nodes that turn on inside this region is represented by μ. We have plotted the node activation probability given by (31), as a function of distance from the transmitter, for $\mu = 10$ and 20 in Fig. 2. We have considered $\lambda = \tilde{\lambda}$ for this plot. From (29), we obtain $R_1 = 0.7071$ for all values of μ and from (30) we numerically compute $R_2 = 1.173, 1.277$ for $\mu = 10$ and 20 respectively. We mention here that we have considered the optimum density of link distance to depend on μ through N as $N = \lceil \mu - 1 \rceil$. Since R_1 does not depend on μ as shown in (29), note that R_2 increases with μ as verified by Fig. 3. This is because the optimum link density guarantees that nodes are now distributed in a wider region. In other words, as μ increases, the activation region increases in area since R_2 increases and R_1 is fixed. Since our objective is to maximize transmission distance, we also observe that as we go farther away from the source, it is more likely that an arbitrary node would get activated.

We have compared the performance of our pseudo-optimum protocol, with a simpler "dumb" protocol that turns on all the nodes around the transmitter upto a fixed distance R_d. For a fair comparison between the two protocols, we limit this transmission radius R_d such that the average number of nodes that turn on within R_d is μ. Hence, we assume the node activation probability $\psi(x) = 1, \forall x \leq R_d$.

We have also plotted the expected value of the maximum transmission distance $\mathbb{E}[V_{max}]$ obtained from (11) in Fig.4 as a function of the number of active receivers μ for $\lambda = 10, 20$ and 30 for both the pseudo-optimum– and dumb protocols. Note that as μ increases, the message travels a greater distance. This is due to the effect of multiuser diversity. From Fig. 4, we find that the expected maximum transmission distance decreases with increase in λ for the dumb protocol. However, in case of the pseudo-optimum protocol the maximum transmission distance is invariant to λ, if $\lambda \geq \tilde{\lambda}$. Recall from Section III-B that we have defined $\tilde{\lambda}$ as the minimum node density that meets the constraint on the average number of active nodes μ. We have marked with arrows in Fig. 4 the points at which λ changes along the plot for the pseudo-optimum protocol, viz. $\tilde{\lambda} < 10$ for $3 \leq \mu \leq 10$. Similarly, $\tilde{\lambda} < 20$ for $3 \leq \mu \leq 20$ and $\tilde{\lambda} < 30$ for $3 \leq \mu \leq 30$.

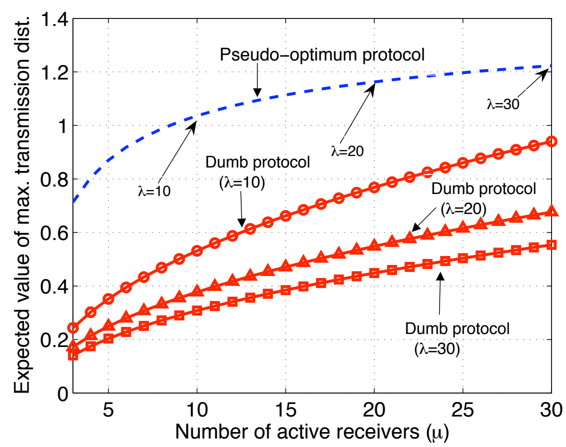

Fig. 4. Expected value of the maximum transmission distance $\mathbb{E}[V_{max}]$ vs. the expected number of active nodes μ.

We have illustrated that geographic transmissions provide us the benefit of higher maximum transmission distance by turning on nodes away from the transmitter. However, this benefit comes at a cost – a considerably higher outage probability. Here, we define outage probability to be the probability that the message remained at the transmitter, or, mathematically, $F_{V_{max}}(0)$. We have simulated the outage probability for the pseudo-optimum protocol. For the dumb protocol, the outage probability expression is obtained from [8]. (Refer to eqn.(8) and the Appendix therein.) As shown in Fig. 5, this cost is quite high when the number of nodes that are awake μ is low. Since the "dumb" protocol is too conservative and turns on nodes close to the transmitter, it suffers lesser outage.

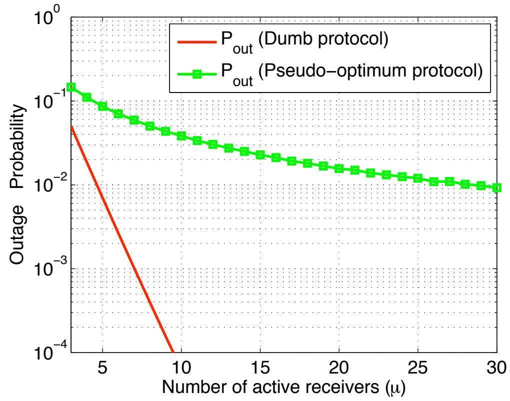

Fig. 5. Outage probability (in logarithmic scale) comparison for the pseudo-optimum – and dumb protocols plotted as a function of the expected number of active nodes μ.

V. Conclusions

In this paper, we have developed a geographic transmission protocol that turns on nodes located inside some region according to a node activation function such that the distance to the farthest successful receiver within this region is maximized. We begin by deriving the optimum density of link distance and then use this optimum density to obtain the activation probability of a node, subject to a constraint on the average number of nodes that are awake. Since our activation function is distance-based, a node can acquire its location information (via G.P.S.) and then compute its distance from the transmitter and hence its activation probability *in situ* and finally decide to turn on/off. We prove mathematically that the node activation region is located away from the transmitter and hence nodes located close to the transmitter can save power by turning off. With a constraint on the average number of nodes that turn on, we find that our protocol activates nodes located further away from the transmitter with higher probability. We also find that the area of the activation region grows with the number of active nodes, viz., we find that doubling the number of active nodes results in 18.5% increase in the area. We point out again that the major contribution in this work is that our node activation is distance-based and hence local in its character. So, apart from providing power savings by turning off nodes, our protocol also consumes less network bandwidth (and hence has a reduced network overhead) because the decision for activation is locally generated.

Appendix

Let G be another distribution function that maximizes (12). Then, for each ν, with $0 < \nu < 1$, we have,

$$\int_0^\infty 1 - \left[1 - \int_t^\infty g(u)\Big(\nu dG(u) + (1-\nu)\, dF(u)\Big)\right]^N dt \leq \int_0^\infty 1 - \left[1 - \int_t^\infty g(u) dF(u)\right]^N dt. \quad (32)$$

I.e. the function of ν on the LHS of (32) is maximized when $\nu = 0$. Consequently, its derivative at $\nu = 0$ is less than or equal to zero. In other words,

$$\int_0^\infty \left[\left[1 - \int_t^\infty g(u) dF(u)\right]^{N-1} \int_t^\infty g(u)\Big(dG\,(u) - dF\,(u)\Big)\right] dt \leq 0.$$

So for all distributions G,

$$\int_0^\infty \left[\left[1 - \int_t^\infty g(u) dF(u)\right]^{N-1} \int_t^\infty g(u) dG\,(u)\right] dt \leq \int_0^\infty \left[\left[1 - \int_t^\infty g(u) dF(u)\right]^{N-1} \int_t^\infty g(u) dF\,(u)\right] dt. \quad (33)$$

Let us denote

$$\mathcal{A} = \left\{ s : g(s) \int_0^s \left[1 - \int_t^\infty g\,(u)\, dF(u)\right]^{N-1} dt = \max_u \left\{ g(u) \int_0^u \left[1 - \int_t^\infty g\,(z)\, dF(z)\right]^{N-1} dt \right\}\right\}.$$

From (33) we find that dF is concentrated on $\mathcal{A}$.

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