EEL 5544 Noise in Linear Systems Lecture 37

MEAN, AUTOCORRELATION, AND AUTOCOVARIANCE FUNCTIONS

- It is generally not practical, and often unnecessary to determine n-dimensional distribution or density functions for RPs
- In many problems, it is sufficient to know about *two-dimensional* statistics and distribution functions

DEFN

> A random process $X(t)$ is a ______________________________ if $E\left\{[X(t)]^2\right\} < \infty$ for each t.

- For second-order RPs, we define the following functions:

DEFN

> The *mean function* (or just *mean*) is given by
>
> $$\mu_X(t) \quad =$$

DEFN

> The *autocorrelation function* is given by
>
> $$R_X(t_1, t_2) \quad =$$

DEFN

> The *autocovariance function* is given by
>
> $$C_X(t_1, t_2) \quad =$$

DEFN

The *normalized autocovariance* of a random process $X(t)$ is given by

$$K_X(t_1, t_2) = \frac{C_X(t_1, t_2)}{\sigma_1 \sigma_2},$$

where $\sigma_i = \sqrt{\text{Var}[X(t_i)]}$.

- Note that

$$-1 \leq K_X(t_1, t_2) \leq 1$$

Interpretations:

- At each t, $\mu_X(t)$ is the expected value of the random variable $X(t)$
  - $\mu_X(t)$ depends only on the one-dimensional density $f_{X,1}(x; t)$

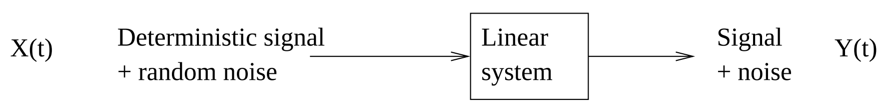
- $R_X(t_1, t_2)$ and $C_X(t_1, t_2)$ measure the statistical "coupling" or "dependence" between $X(t_1)$ and $X(t_2)$
  - $R_X(t_1, t_2)$ and $C_X(t_1, t_2)$ depend only on the one- and two-dimensional densities
  - if $X(t_1)$ and $X(t_2)$ are statistically independent, then $C_X(t_1, t_2) = 0$
  - if $C_X(t_1, t_2) = 0$, then $X(t_1)$ and $X(t_2)$ are uncorrelated

- Note: generally only need to know one of C_X and R_X

$$C_X(t_1, t_2) =$$

Given $X(t)$ with finite power ($X(t)$ a second-order RP), are μ_X, R_X, and C_X all finite?

Application preview: Noise in Linear Systems



- Define the (normalized) power as the power into a 1 Ohm resistor.
- Then for a random signal $X(t)$, regardless of whether $X(t)$ is a voltage or amplitude signal, the power is $E[X^2(t)]$.
- What is the noise power at input and output?
  - Input: Noise power = $E\left\{[X(t)]^2\right\} = R_X(t, t)$
  - Output: Noise power = function of $R_X(t_1, t_2)$ and **not** just $R_X(t, t)$

Example